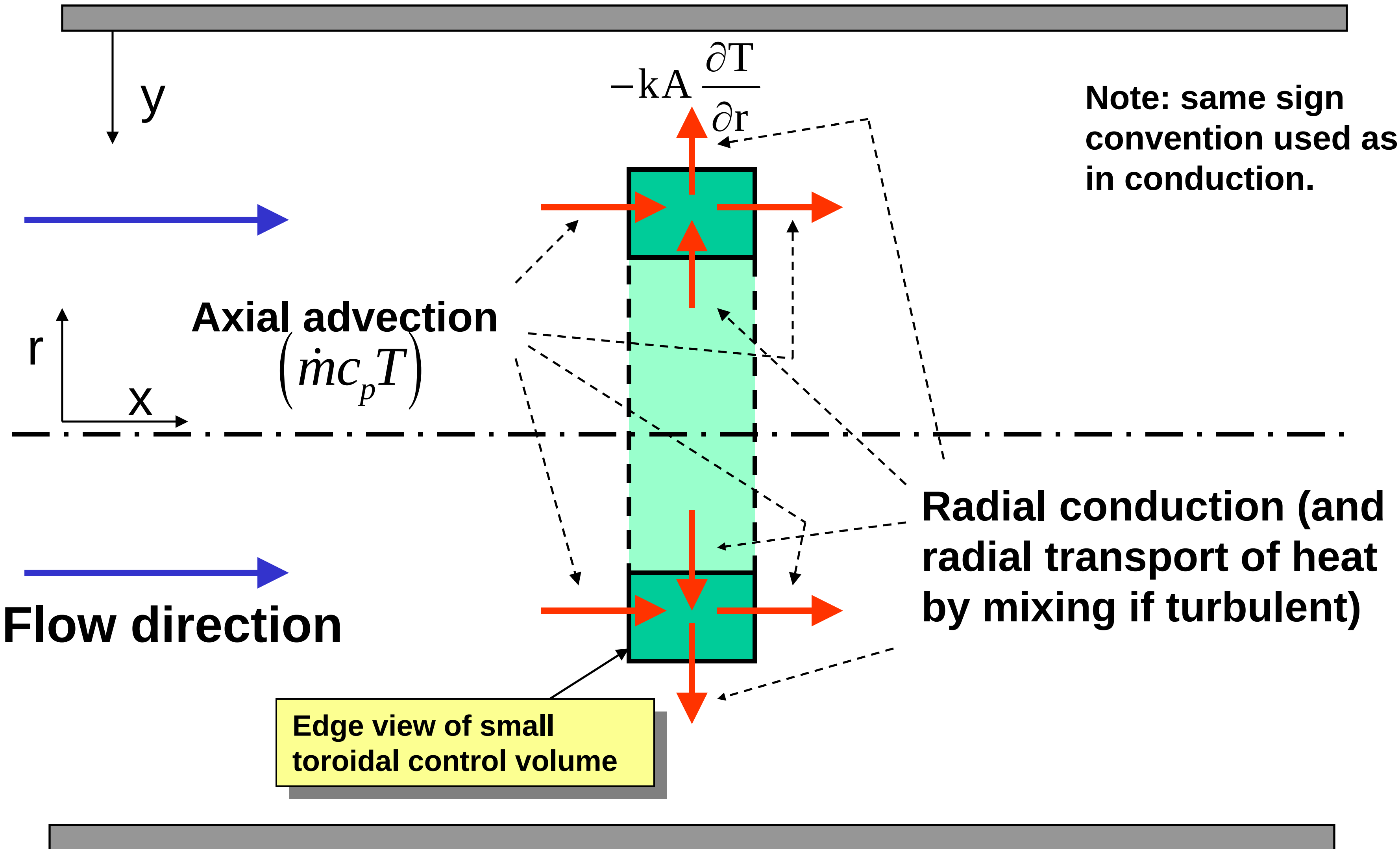
# Modeling Internal Flows with the HTT_pipe Module

- Modeling heat transfer in internal flows
- Finding h, the convective heat transfer coefficient*
- Interpreting results from studio internal flows module (HTT_pipe)

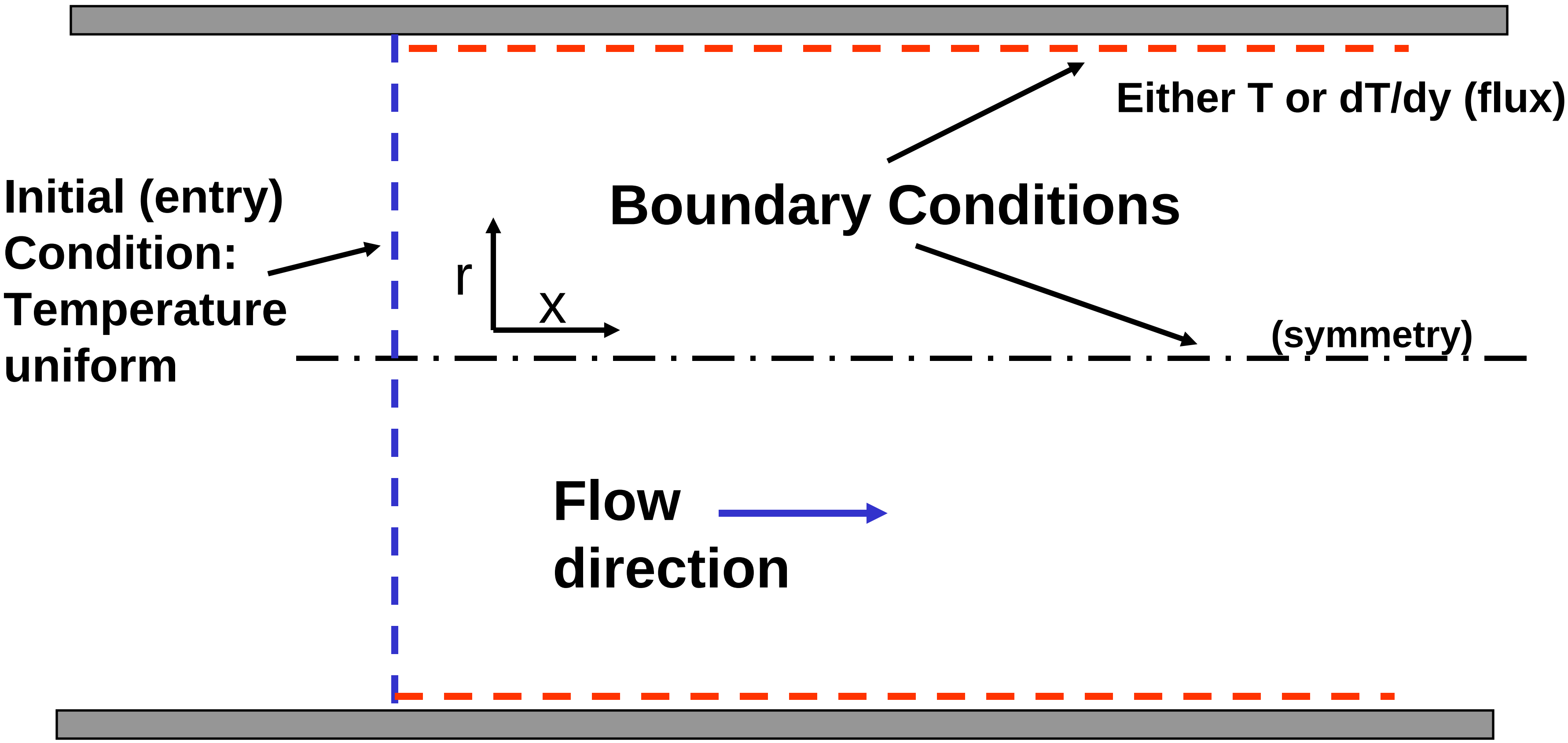
*BUT NOTE WELL: You may not even need the convection coefficient except for use in verifying the module results. You can find the heat transferred from the change in the mean temperature of the fluid.

# The Physics Included in Module:




In – out = 0 gives predictive equation for temperature.

# The Numerical Procedures inside Module:




**March downstream filling in unknown interior values using the heat balance equation based on previous slide. Very similar to transient conduction except now marching out in space (downstream) rather than in time. (Parabolic PDE)**

# The Numerical Scheme Used in this Module is Called the *Finite-Volume* Method

- **The system of linear equations solved at each axial station along the pipe come directly from the conservation of energy statement applied to a representative control volume.**
- **You are approximately solving a Partial Differential Equation, but doing it numerically rather than analytically.**
  - **To do such a calculation yourself, you do not need to know much about PDE's, but you must understand the physics!**
- **This FV method is distinct from the *Finite Element* Method (FE or FEM), which is also used for PDE's and also involves the discretization of the solution domain.**

# The Model Used Here Solves Only the Energy Transport Equation

- **The Energy Equation was derived in the Analysis group of Slides:**

$$\underbrace{\rho c_p u \frac{\partial T}{\partial x}}_{\text{axial advection}} = \underbrace{\frac{k}{r}\frac{\partial}{\partial r} r \frac{\partial T}{\partial r}}_{\text{conduction in radial direction}}$$

- **If we wanted to include the temperature dependence of properties on the velocity profile or to consider combined entry length problems, we would have to solve a momentum (and continuity) equation in conjunction with the energy equation.**
- **The equation above is not really different from that solved for transient, one-dimensional conduction except that:**
  - **velocity (u) is a function of radius**
  - **we march downstream (in x) rather than outward in time.**

# For $Re_D$ > 2300 a Turbulence Model Is Automatically Activated

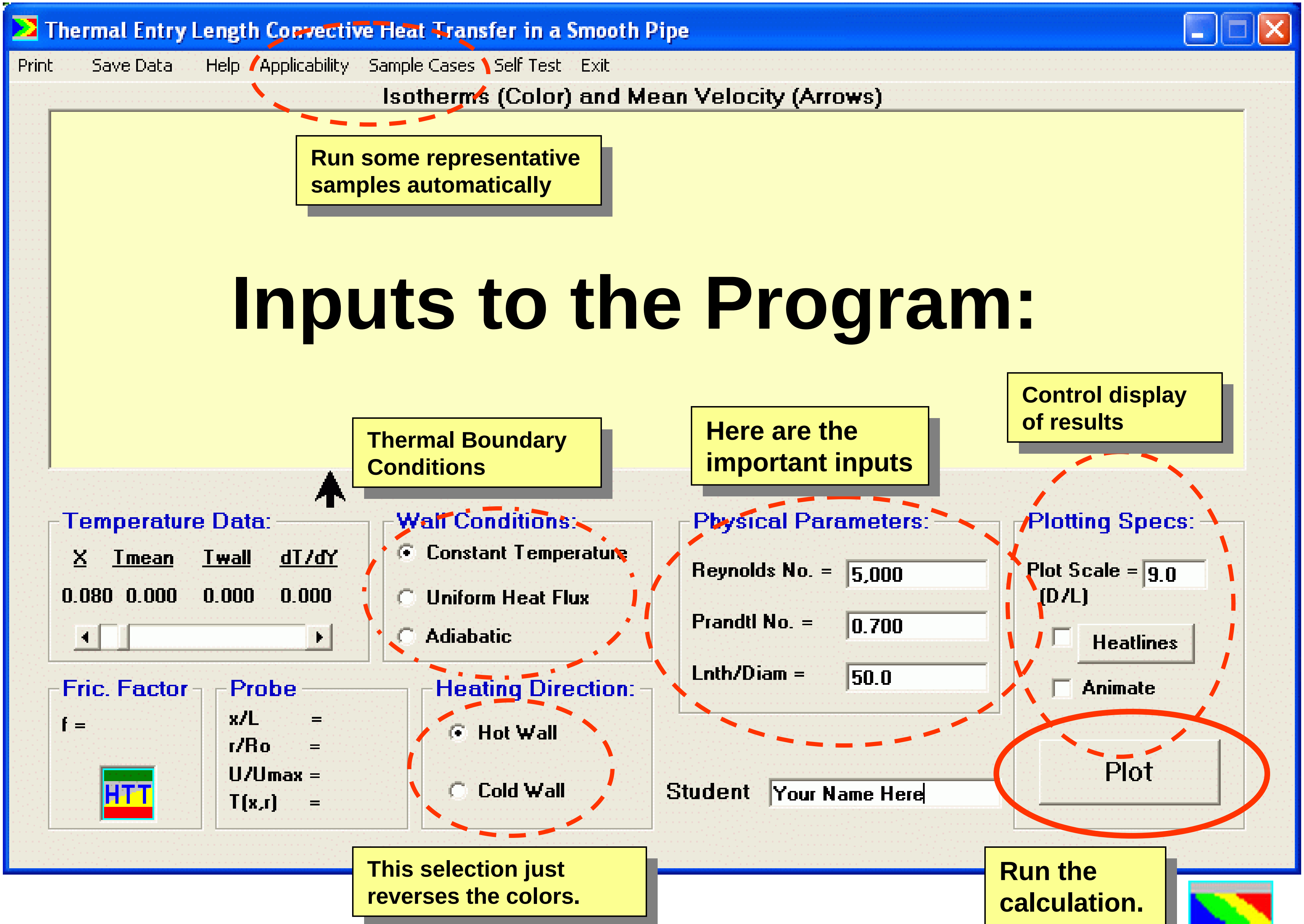
- **Turbulence in effect adds to the cross-stream (radial) transport of heat:**

$$\rho c_p u \frac{\partial T}{\partial x} = \frac{k}{r}\frac{\partial}{\partial y}\left[ r\left(1+\underbrace{\frac{\mathrm{Pr}}{\mathrm{Pr}_t}\varepsilon_m^+}_{\text{effect of turbulence}}\right)\frac{\partial T}{\partial y}\right]$$

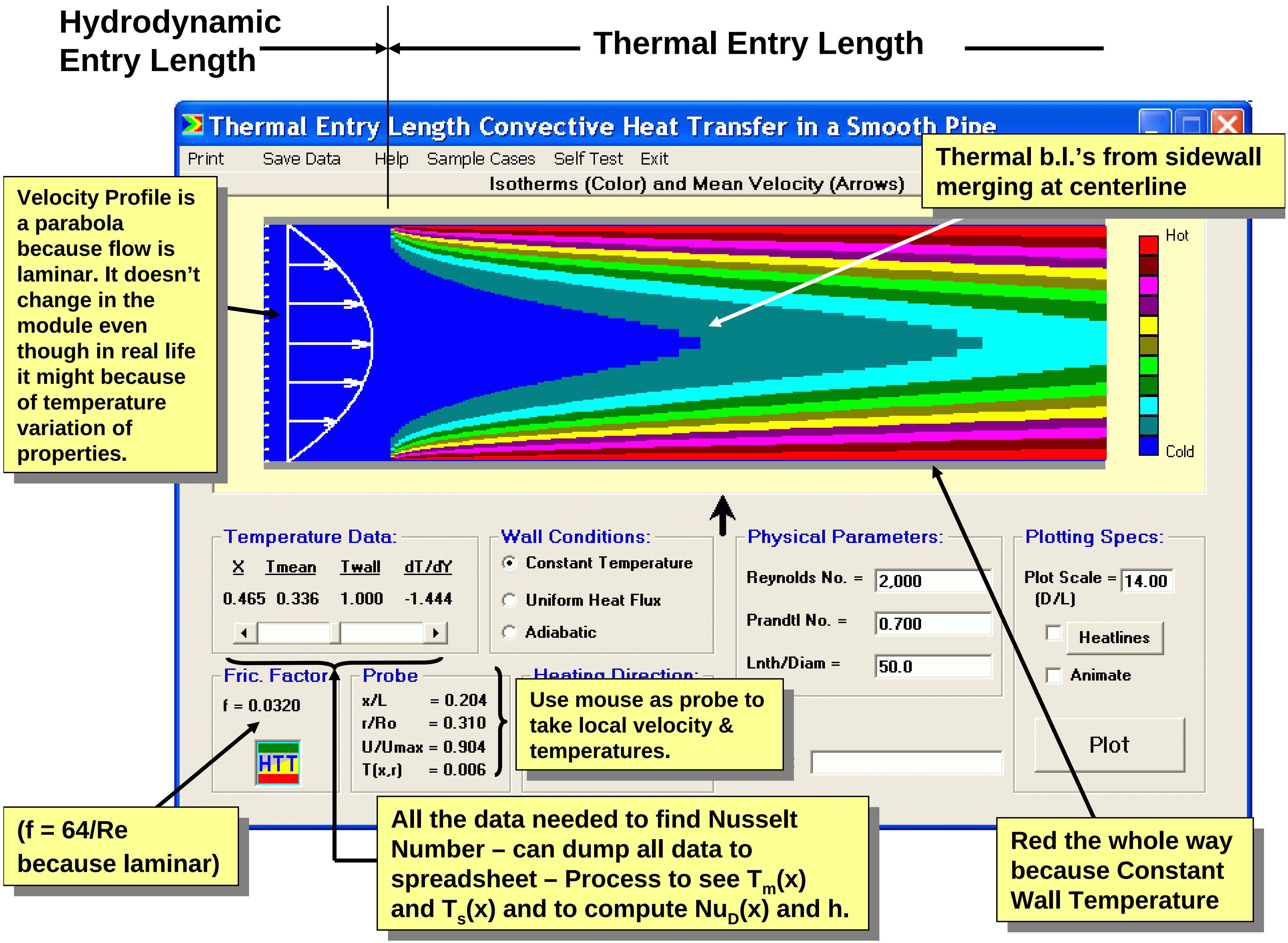
- **A simple *mixing length* turbulence model that includes a van Driest damping factor has been implemented.**
- **See books by Cebeci and Bradshaw, Schetz, etc. for details.**

# Turbulence Model – Additional Notes

- **Since the turbulence model uses the friction factor as its starting point, the good agreement coming from this heat transfer model when compared with, e.g., the Gnielinski convection correlation is not surprising.**
- **In the simple "animation" scheme included in the module the "orbit" of the particles is taken as dependent on the mixing length as that particular radius.**
- **The model used here only addresses smooth pipes. Heat transfer enhancement schemes for internal flows generally involve increasing the cross-stream mixing using inserts of one sort or another while adding as little additional friction as possible.**



Thermal Entry Length Convective Heat Transfer in a Smooth Pipe
Print Save Data Help Applicability Sample Cases Self Test Exit
Isotherms (Color) and Mean Velocity (Arrows)
Run some representative samples automatically
Inputs to the Program:
Thermal Boundary Conditions
Here are the important inputs
Control display of results
Temperature Data:
X Tmean Twall dT/dY
0.080 0.000 0.000 0.000
Wall Conditions:
Constant Temperature
Uniform Heat Flux
Adiabatic
Physical Parameters:
Reynolds No. = 5,000
Prandtl No. = 0.700
Lnth/Diam = 50.0
Plotting Specs:
Plot Scale = 9.0
(D/L)
Heatlines
Animate
Fric. Factor
f =
HTT
Probe
x/L =
r/Ro =
U/Umax =
T(x,r) =
Heating Direction:
Hot Wall
Cold Wall
Student
Your Name Here
Plot
This selection just reverses the colors.
Run the calculation.


Hydrodynamic Entry Length
Thermal Entry Length
Thermal Entry Length Convective Heat Transfer in a Smooth Pipe
Print
Save Data
Help
Sample Cases
Self Test
Exit
Isotherms (Color) and Mean Velocity (Arrows)
Thermal b.l.'s from sidewall merging at centerline
Velocity Profile is a parabola because flow is laminar. It doesn't change in the module even though in real life it might because of temperature variation of properties.
Hot
Cold
Temperature Data:
X
Tmean
Twall
dT/dY
0.465
0.336
1.000
-1.444
Wall Conditions:
Constant Temperature
Uniform Heat Flux
Adiabatic
Physical Parameters:
Reynolds No. = 2,000
Prandtl No. = 0.700
Lnth/Diam = 50.0
Plotting Specs:
Plot Scale = 14.00
(D/L)
Heatlines
Animate
Plot
Fric. Factor
f = 0.0320
HTT
Probe
x/L = 0.204
r/Ro = 0.310
U/Umax = 0.904
T(x,r) = 0.006
Heating Direction:
Use mouse as probe to take local velocity & temperatures.
(f = 64/Re because laminar)
All the data needed to find Nusselt Number – can dump all data to spreadsheet – Process to see $T_m(x)$ and $T_s(x)$ and to compute $Nu_D(x)$ and h.
Red the whole way because Constant Wall Temperature

# Finding the Nusselt Number from analysis, modeling or experiment:

$$q'' = h\,(t_s - t_m) = k_f \left.\frac{\partial t}{\partial r}\right|_{r=r_o} = -k_f \left.\frac{\partial t}{\partial y}\right|_{y=0}$$

**Wall temp.**

**(y is measured inward from surface)**

$$T \equiv \frac{t - t_i}{t_s - t_i}$$

Inlet temp.

Surface temp.

**Substitute in above:** $h\,(T_s - T_m) = \dfrac{k_f}{r_o} \left.\dfrac{\partial T}{\partial r}\right|_{\text{surface}}$

**(r is non-dimensionalized by $r_o$, radius of pipe )**

# Finding the Nusselt Number (cont.)

**Rearranging:**

$$\frac{hD}{k_f} \equiv Nu_D = \frac{2\left.\dfrac{\partial T}{\partial r}\right|_{surface}}{T_s - T_m} = \frac{-2\left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_m}$$

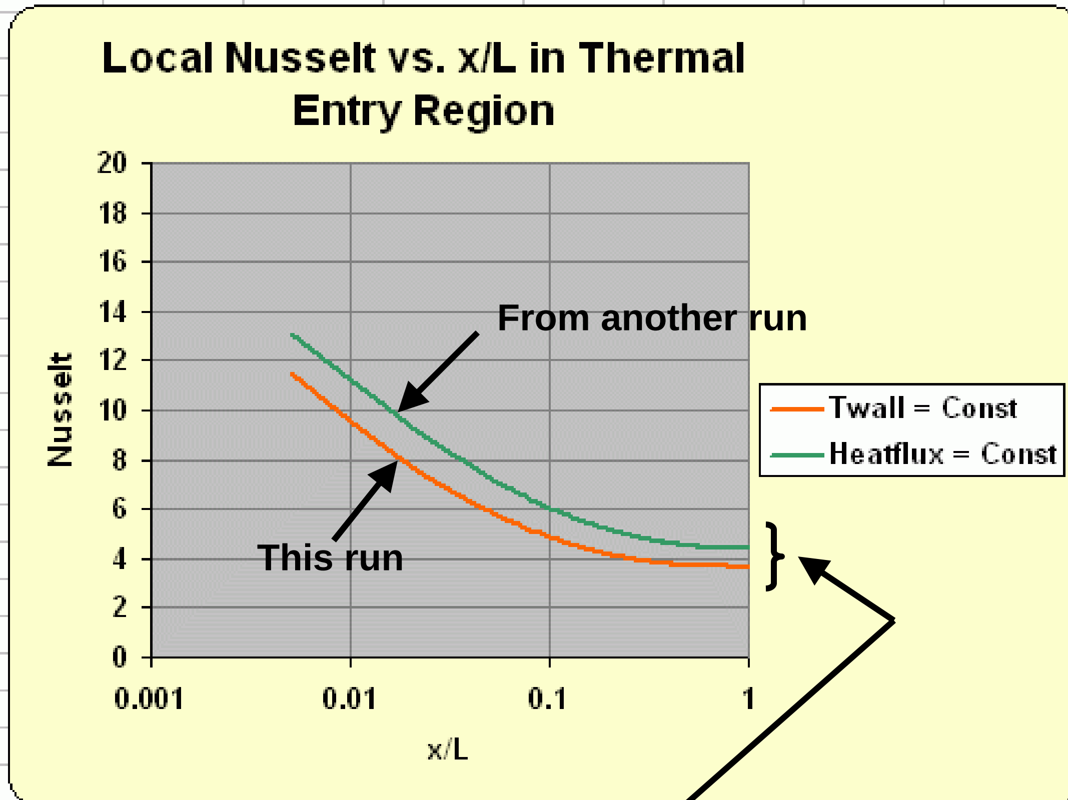
**So if you have the surface temperature, mean fluid temperature and the temperature gradient at the surface*, you're all set! Get them from analysis (laminar flows), experiments (laminar and turbulent) or modeling (the pipe flow module)**

**(For external flows we just needed temperature gradient at the surface (for a constant surface temperature case) because the freestream temperature wasn't changing.)**

# Processing Data (for two laminar, thermal entry length problems)

| | | | | |
|---|---|---|---|---|
| Output Data for Flow in Heated Pipe | | | | |
| 6/9/2005 | 11:37:07 AM | | | |
| Wall Conditio | Constant Temperature | | | |
| Heat Directior | Hot Wall | | | |
| | | | | |
| Reynolds No: | 1888 | | | |
| Prandtl No: | 0.707000017 | | | |
| L/D: | 140 | | | |
| | | | | |
| XX | Tmean | Twall | dT/dy | $Nu_D$ |
| 0 | 0.035328526 | 1.000002 | -8.44213 | 17.50256 |
| 0.005 | 0.058012139 | 1 | -5.42048 | 11.50861 |
| 0.01 | 0.076465853 | 1.000004 | -4.40976 | 9.5497 |
| 0.015 | 0.092565566 | 1.000002 | -3.84722 | 8.479319 |
| 0.02 | 0.107093617 | 1.000006 | -3.47166 | 7.776044 |
| 0.025 | 0.12046735 | 1.000007 | -3.19584 | 7.267062 |
| 0.03 | 0.132941887 | 1.000005 | -2.98095 | 6.875971 |
| 0.035 | 0.144687667 | 1.00001 | -2.80684 | 6.563236 |
| 0.04 | 0.155825049 | 1.000004 | -2.66142 | 6.305352 |
| 0.045 | 0.16644384 | 1.000014 | -2.53754 | 6.088365 |
| 0.05 | 0.176612094 | 1.000008 | -2.42985 | 5.902031 |
| 0.055 | 0.186384276 | 1.000011 | -2.33522 | 5.740285 |
| 0.06 | 0.19580394 | 1.000015 | -2.25096 | 5.597944 |
| 0.065 | 0.204906181 | 1.000002 | -2.17512 | 5.471346 |
| 0.07 | 0.21372135 | 0.999998 | -2.10652 | 5.358216 |
| | | | | 56247 |
| | | | | 3589 |
| | | | | 9694 |
| | | | | 0212 |
| | | | | 0729 |
| | | | | 5511 |

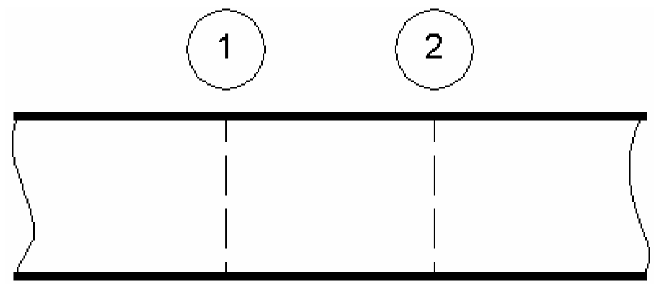
**Using formula derived on previous slide**




**The Reynolds number corresponds to air at a mean temperature of 300K flowing at 1 m/s in a .03m diameter pipe. The criterion for a fully-thermally developed flow gives L/D = Re*Pr/20 = 67.**

**Note asymptotic values: 3.66 for Twall = Const and 4.36 for Heat flux = Const.**

# 2nd Method: Control Volume Energy Balance




**Energy passing Station 1 + that added between 1 & 2 = Energy passing Station 2.**

$$\dot{m} c_p t_{m1} + h(x) 2\pi r_o \Delta L (t_s - \bar{t}_m) = \dot{m} c_p t_{m2}$$

$$\dot{m} = \rho \overline{V} \pi r_o^2$$

Rearrange to give:

$$\frac{h}{\rho \overline{V} c_p} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} \equiv St = \frac{r_o (T_{m2} - T_{m1})}{2\Delta L\,(T_s - \overline{T}_m)}$$

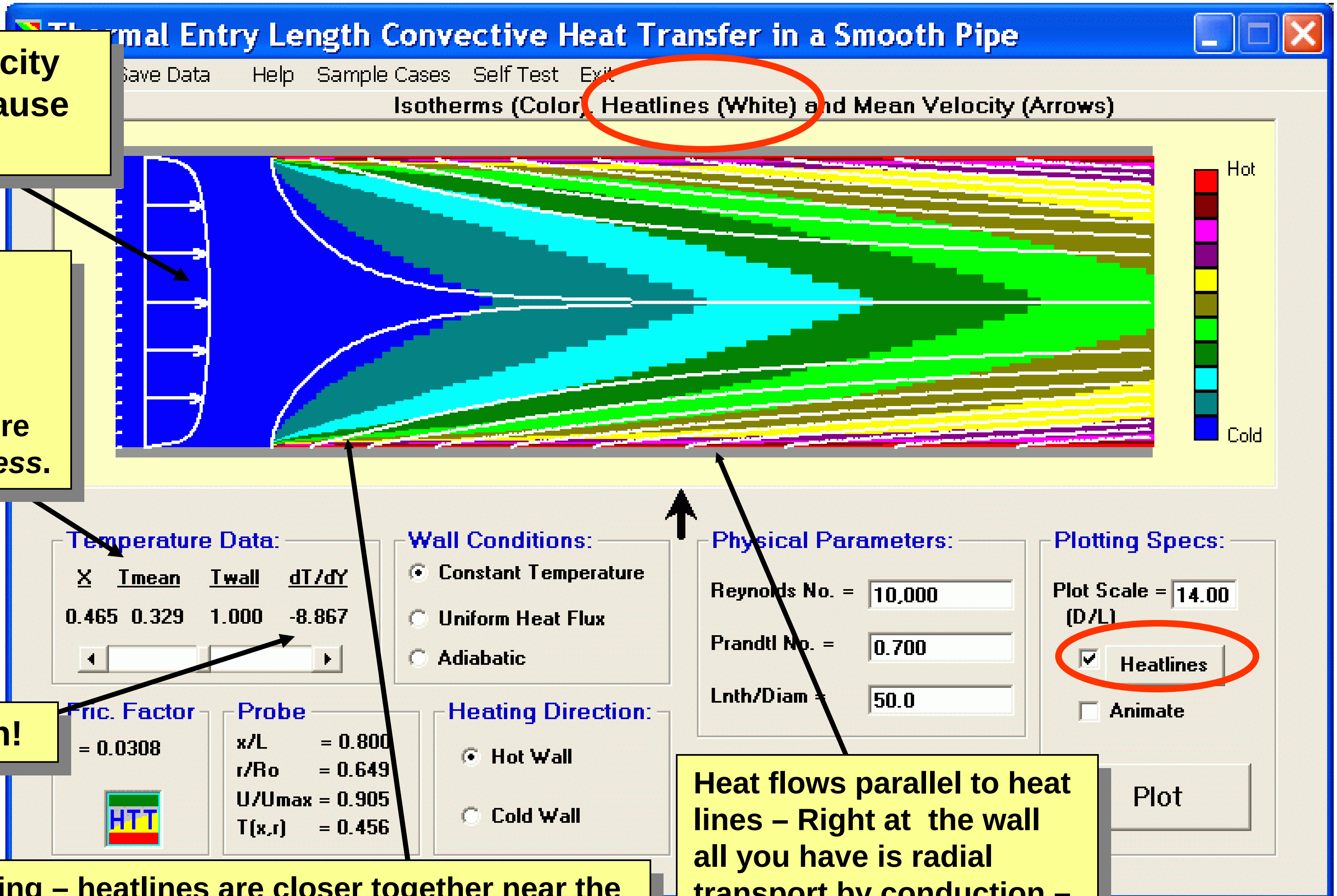
**Stanton Number**

**With this method you don’t need surface temperature gradient. Easy to do in lab.**

# Heatlines:

- With <u>conduction</u> only we talked about isotherms and heatflux or heatflow lines.
  - Heat flow was normal to isotherms and parallel to heat flux lines.
- With convection we have another heat flow mechanism besides conduction.
  - Isotherms alone are misleading.
  - Use *heat flow lines* to show path of heat.
    - Total heat flow (including conduction and advection (transport by moving fluid) is parallel to heat flow lines.
  - Analogous to streamlines in fluid mechanics
  - Shown in module as white lines superimposed on isotherms.

# At a higher Reynolds number and with *heatlines* plotted in addition to isotherms.




**Flatter velocity profile because turbulent**

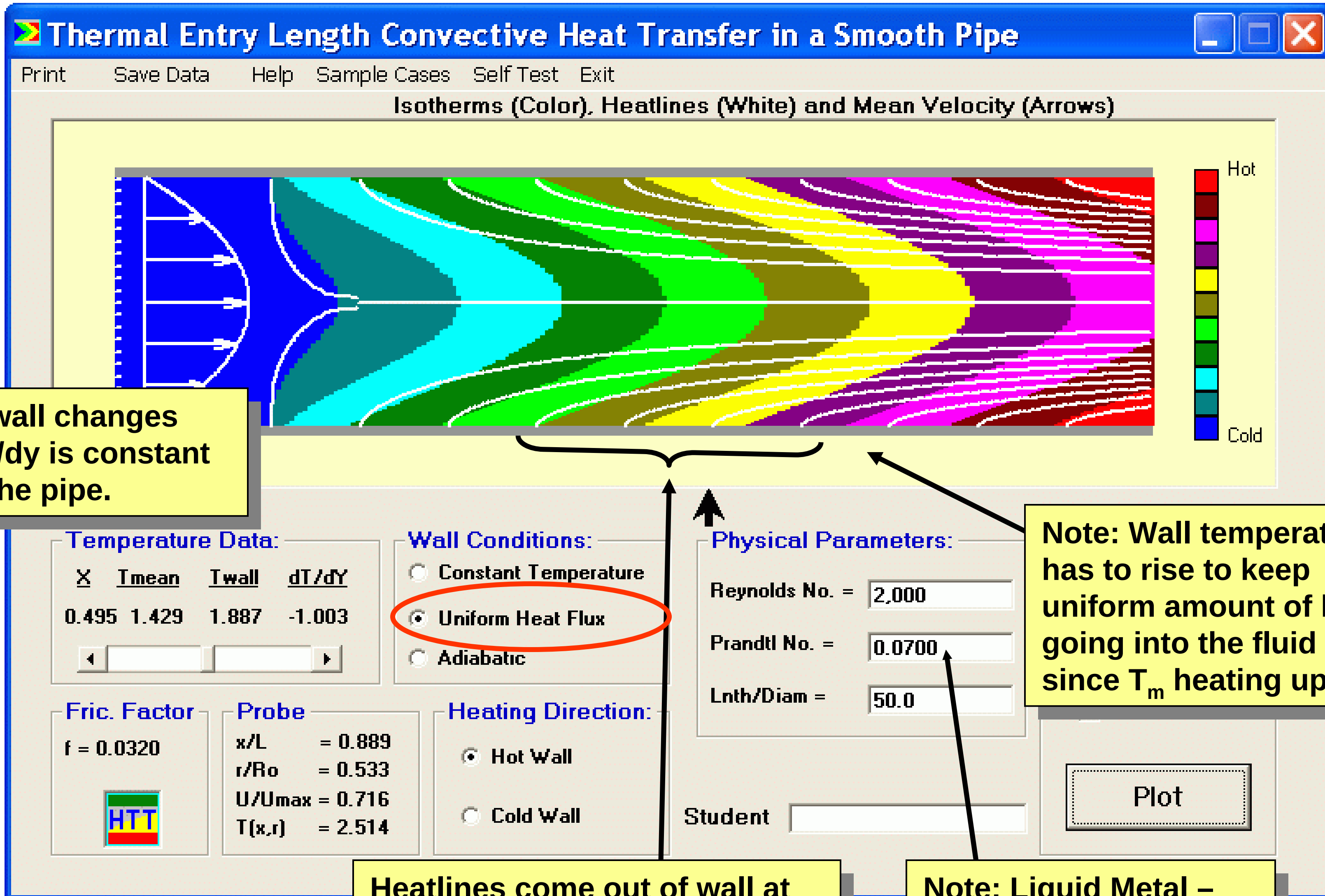
**For constant wall temperature, Tmean is a direct measure of *effectiveness*.**

**Note sign!**

**Note spacing – heatlines are closer together near the start of the heated section (indicating more heat transfer) for two reasons: ($T_s$-$T_m$) is biggest there and so is h.**

**Heat flows parallel to heat lines – Right at the wall all you have is radial transport by conduction – so they come in normal to surface**

# Constant Heat Flux Rather Than Constant Wall Temperature




**Now Twall changes and dT/dy is constant along the pipe.**

**Note: Wall temperature has to rise to keep uniform amount of heat going into the fluid – since $T_m$ heating up.**

**Heatlines come out of wall at equal intervals corresponding to uniform heat flux**

**Note: Liquid Metal – great cross-stream conduction – and super heat transfer!**

Applicability of the HTT_pipe model

# Applicability of the HTT_pipe Internal Flows Module

| | Laminar $Re_D$ <~ 2300 | Transition 2300 < $Re_D$ < 10000 | Turbulent $Re_D$ >~ 10,000 |
|---|---|---|---|
| Fully-developed velocity (fdv), fully-developed temperature (fdt) criteria: | $(x/D)_{fdv}$ ~ Re/20 $(x/D)_{fdt}$ ~ RePr/20 | | $(x/D)_{fdv}$ ~ 10 - 60 $(x/D)_{fdt}$ ~ 10 - 60 |
| Thermal Entry Length (velocity profile already fully developed when heating starts) | Yes T | Yes | Yes T H |
| Fully-Developed (both velocity and temperature profiles fully-developed) | Yes (asymptotic value) T H | Yes (asymptotic value) T H | Yes (asymptotic value) L ,T L , H |
| Combined Entry Length (neither fully-developed) | No (but note that for Pr >> 1, fdt>> fdv) T | No | No (but note that fdv and fdv are both short) |
| Large Fluid Property Variations | No | No | No |

T = Constant Wall Temperature
H = Constant Surface Heat Flux
L = Liquid Metal

Close

This form in the HTT_pipe module will launch sample runs of the cases shown .