Sandwich Wall (ICF) Construction

Introduction

Figure 1Sandwich Wall under Construction

Recently an alumnus designing his own dream home decided to use what he had learned about thermal storage from having done the Monticello project as a student. The cost of the materials and labor involved in building 16" thick brick walls like those at Monticello would, of course, be prohibitive today.   On the other hand, since Jefferson's time we have recognized the insulating value of air immobilized in fiberglass and other lightweight insulations.   This new home, shown under construction in the photo, uses a construction that combines the advantages of concrete as a thermal storage medium with lightweight Styrofoam[tm] as an insulating material. The format is known at Insulated Concrete Forms (ICF) construction.

In this construction rigid closed cell, extruded polystyrene foam boards are used as the forms when the concrete is poured, but unlike normal plywood or steel forms, which are removed for reuse elsewhere when the concrete has hardened, the foam board is left there permanently.   The final construction thus consists of two layers of lightweight foam, each having a significant insulating ("R") value, plus a layer of concrete having a large thermal mass in the space between them.   In the basement the foam layers are 2.375″ thick and the concrete is 7.875″ thick, while in the living space 6.25″ of concrete is sandwiched between 2.625″ thick sheets of foam. A total of 136 cubic yards of concrete were used in the 4400 sq. ft. (finished basement and main level) home.   Standard surface treatments are used inside and out.

In our formulation of this problem, we will ignore the steel "rebar" that is seen protruding from the concrete in the photo. The braces seen above on the inside are shipped back to the supplier of the kit once the concrete work is complete.   The black vertical stripes seen on the foam are nailing (furring) strips and are on the outside as well, allowing the builder to finish both the inside and outside surfaces as they so choose.   Other than the deeper than usual window and door penetrations, the finished construction looks quite conventional.
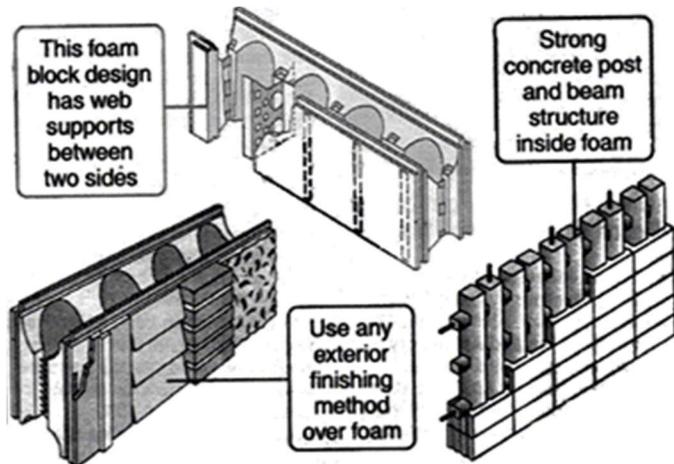

Figure 2 Alternate foam/concrete construction [1]

The foam-concrete-foam configuration seen in the photograph above lends itself readily to a simple one-dimensional analysis; other designs use the same principle of combining the thermal capacity of concrete with the insulating value of polystyrene foam. The analysis of a system such as that shown to the right as one-dimensional would, however, be problematic.

Another contemporary construction, autoclaved aerated concrete, combines the insulating value of trapped air more directly with the heat capacity of masonry [2, 3]. These blocks are pre-cast at the factory, where a chemical reaction that takes place before the concrete has set creates thousands of tiny bubbles throughout the material. These lightweight blocks have been used elsewhere, particularly in Europe, for over half a century and are now becoming more popular in residential and commercial construction in the southern U.S.

Implementation

In this project we will develop a 1-D, finite-difference (volume) model to study the transient storage and heat transmission effects in the Styrofoam/concrete composite wall. We will use a grid spacing of 0.5″ (.0127m) and approximate thickness values in the numerical implementation in order to simplify the nodalization. Let us assume that each piece of Styrofoam is 2 ¾″ thick so that we have 5 ½ nodes in the insulation. Assuming that the concrete is 6 ½ ″ thick, then we have 13 full cells in the middle layer. The entire computing region will then have 23 full cells, plus half-cells at each end, giving a total of 25 unknown temperatures:

We will treat the surface boundary conditions the same way as in the earlier Monticello
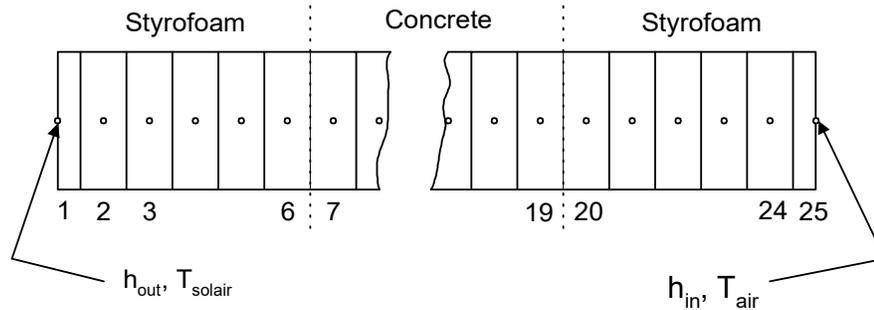

Figure 3Nodalization for finite-difference (volume) calculation

problem. At the inside surface let us assume h_{in} = 8 W/m²·K with a constant air temperature of 21ºC. For the outside we must include both radiative and convective gains and losses. Rather than treat these gains and losses separately, it is common (and vastly easier) to use an effective temperature, which when put into the standard Newton's law of cooling (q = h A Δ T) has the equivalent effect. The *Sol-Air* temperature is a function of the date, hour, latitude, and the radiative properties and orientation of the wall with respect to the sun. We will use values for a dark, west-facing wall at 40º N latitude on July 21. Hour-by-hour T_{solair} values for these conditions as tabulated in the ASHRAE Handbook of Fundamentals [4] are as follows:

> 24.4, 24.4, 23.8, 23.3, 23.3, 25.0, 27.7, 30.0, 32.7, 35.0, 37.7, 40.0,
> 53.3, 64.4, 72.7, 75.5, 72.2, 58.8, 30.5, 29.4, 28.3, 27.2, 26.1, 25.0

Here the first value (24.4ºC) corresponds to 1:00 a.m.; the last value (25.0ºC) corresponds to midnight. Note that the effective Sol-Air temperature gets up to nearly 170ºF (75.5ºC) in the afternoon. (We are assuming there are no trees shading this wall.) If you wind up using a timestep less than an hour, it would be advisable to fill in the intermediate values using linear interpolation. Otherwise the heat flux vs. time plot you will eventually obtain will exhibit some non-physical "kinks" in response to the step change in boundary condition every hour. Use a convection coefficient of h_{out} = 18 W/m²·K at the outside surface. (The higher outdoor value reflects the effect of breezes that are presumably not present indoors.)

At the end you will want to plot the heat transfer at both the inner and outer surfaces of the wall. This means that in order to make the plot, you will need the predicted temperature of the outside surface and that of the inside surface for each hour of the last day of your simulation. The heat transferred at the inner wall divided by the overall heat transfer coefficient for the wall will give you the hourly "cooling load temperature difference"(CLTD) commonly used in load estimating.

Numerical Formulation

We will set this problem up using an *implicit* formulation for several reasons: (1) we will want to run this simulation over a fairly long period of time, (2) we have the two internal material interfaces to handle, and (3) we have the convective boundary conditions at both ends. With the implicit solution we will be able to use relatively long time steps, in fact, a full hour will be very convenient.

Let us consider a generic internal cell first. For such a cell the discretized heat balance equation written in an implicit form reads:

$$\left(\rho c_p\right)_i \frac{T_i'-T_i}{\Delta t}\Delta x = -k_l \frac{T_i'-T_{i-1}'}{\Delta x} - (-)k_r \frac{T_{i+1}'-T_i'}{\Delta x} \qquad (1)$$

Here the primes indicate the advanced (new) time value. Normally we would divide through by the density-specific heat product, assume k is uniform and introduce a grid Fourier number,

$$Fo = \frac{k}{\rho c_p} \frac{\Delta t}{(\Delta x)^2} = \alpha \frac{\Delta t}{(\Delta x)^2} \; . \qquad (2)$$

Here, although the density-specific heat product for each cell is uniform, we need to be able to handle different thermal conductivities for the inward flux at the left and the outward flux to the right (specifically for nodes 6,7,19 and 20 which are adjacent to the material interfaces). That being the case, here instead we divide through and introduce the coefficients Cl_i and Cr_i (which for most cells are simply the grid Fourier number):

$$T_i' = T_i + Cl_i\left(T_{i-1}'-T_i'\right) + Cr_i\left(T_{i+1}'-T_i'\right) \qquad (3)$$

Now solve for T_i' :

$$T_i' = \left[T_i + Cl_i T_{i-1}' + CR_i T_{i+1}'\right] / \left[1 + Cl_i + Cr_i\right] \qquad (4)$$

Thinking now in the context of a spreadsheet with the spatial coordinate (x) running across and time increasing downward, we have a cell formula relating the temperature at a point with the (known) temperature above it (T_i) and the unknown temperatures to its left (T_{i-1}') and right (T_{i+1}').

When one derives the appropriate form of the heat balance equation for the half cell at the left exposed to convection (Cell #1), one realizes that it can also be put in the form of Equation 3, but with T_{i-1}' replaced with the known T_{solair}. The same principle applies at the interior surface. Thus, the same cell formula may be used for all cells. Most work is involved in determining the

proper coefficients Cl and Cr for all 25 cells, including the generic internal cells, the ends, and interfaces. These coefficients may be pre-computed and stored in two rows at the head of each of the 25 temperature columns. Don't forget how you must address these values in your cell formulae – one direction will have a relative address and the other an absolute address.

During the early development of your spreadsheet you might want to concentrate your efforts on the internal cells and, only once they appear to be computed correctly, moving on to the surface cells. Temporarily setting the outside surface cells to $T_{solair}(t)$ and the inside surface temperature to the inside air temperature makes a good strategy.

Additional Spreadsheet Notes

With cell formulae like that in Eqn. 4 above, your spreadsheet will give an error message informing you that you have a "circular reference." Indeed, what you are really trying to solve here is a *tridiagonal* system of linear equations at each time step (Δt). If you were to write out the complete system of 25 equations in 25 unknown temperatures that you need to solve at each time step, you would soon realize that each unknown temperature depends only on its immediate neighbors. Written in matrix form $\mathbf{Ax = B,}$ the coefficient matrix $\mathbf{A}$ would have a bandwidth of three. You could implement a direct solution technique for a tridiagonal system based on the well-known Thomas algorithm [5] in only about 15 lines of code, but here we'll have the spreadsheet solve them approximately using iteration.

You can wait for your spreadsheet to tell you that you have a "circular reference" (meaning that you have cell formulae that depend on other unknown values) and follow its directions as to what to do about it, or you can take a proactive approach and specify that it should iterate. You can specify how many iterations to allow and how tight a tolerance to use. Don't use a huge number of iterations at first, nor an excessively small tolerance. Later, after you have gotten this project working you will want to crank up the number of iterations (into the thousands) and tighten up the tolerance (e.g., to .00001). Ideally you would solve the tri-diagonal system completely at each timestep before moving on to the next, but unfortunately some spreadsheets (including Excel) don't give you any control over the direction of the iteration that it will do for you. If your spreadsheet does give you the option, you will want to iterate thoroughly over each row before moving down to the next one.)

When we did the original Monticello problem (17" of solid brick), we simply started the calculation from a guessed initial state and ran it through four or five days to get to a fairly steady periodic state. We copied a single day's worth of cell formulae (24 rows) and boundary conditions and pasted it several times at the bottom. With the configuration used in the current problem, it takes a fairly long time to reach a steady periodic state (physically), plus Excel does its iteration in both directions. So that approach seems pretty unwieldy. Since we are mainly interested in the steady, periodic condition, we recommend that you insert another row just above your 1:00 a.m. row. For that row you'll specify that it be equal to the value it computes for that cell at midnight (down at the bottom). Eventually after enough iteration you'll have a 24-hour period of the steady, periodic solution you're looking for – and the entire spreadsheet, including headings, will have less than about 30 rows. The downside is that you will not have a feeling for how long (in days) it takes for this construction to reach a steady, periodic condition.

One way of watching this calculation converge is to set up columns where you will compute the heat fluxes at the inner and outer surfaces. When it has finally converged, those two

fluxes summed over 24 hours ought to be equal – and when you see that you will have a lot of confidence in your solution!

References

1. Dulley, James T., "Concrete/foam block construction combines stability and efficiency," *The Daily Progress,* Charlottesville, VA, Mar. 23, 2000.

2. Jardine, K. and Cameron, J., "Building for Affordability and Energy Efficiency," *Fine Homebuilding*, Spring/Summer 1999, pp. 82-87

3. Bukoski, S.C., "Autoclaved Aerated Concrete: Shaping the Evolution of Residential Construction in the United States," Construction Engineering and Management Program, Georgia Institute of Technology, August 14, 1998.

4. *1997 ASHRAE Handbook - Fundamentals*, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta (1997).

5. Chapra, S.C., and Canale, R.P, *Numerical Methods for Engineers*, 2nd Ed., McGraw-Hill, New York, 1988.

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