

HYDROLOGIC FORECAST PRODUCTS from BAYESIAN FORECASTING SYSTEM

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BAYESIAN FORECASTING THEORY

The Bayesian theory (Krzysztofowicz, 1999) provides a general methodological framework for probabilistic forecasting via any deterministic hydrologic model.

Within this framework, a variety of Bayesian Forecasting Systems (BFSs) suited to different purposes can be developed.

The first prototype systems were developed to produce short-term probabilistic forecasts of river stages, stage transitions, and floods based on probabilistic quantitative precipitation forecasts (PQPFs).

Herein we show examples of probabilistic hydrologic forecast products.

NOTATION

n — index of times

H_n — predictand: actual river stage

h_n — realization of H_n

CASE STUDY

Forecast point: Eldred, Pennsylvania
Allegheny River

Drainage area: 550 miles² (1430 km²)

PQPF: probability of precipitation occurrence in 24-h
distribution of 24-h basin average amount | occurrence
expected disaggregation of amount into 6-h subperiods

PROBABILISTIC FORECAST PRODUCTS

PRSF – Probabilistic River Stage Forecast

$$\Psi_n(h_n) = P(H_n \leq h_n) \quad n = 1, \dots, N$$

PSTF – Probabilistic Stage Transition Forecast (Markov)

$$\Theta_n^*(h_n|h_{n-1}) = P(H_n \leq h_n | H_{n-1} = h_{n-1}) \quad n = 1, \dots, N$$

PFF – Probabilistic Flood Forecast

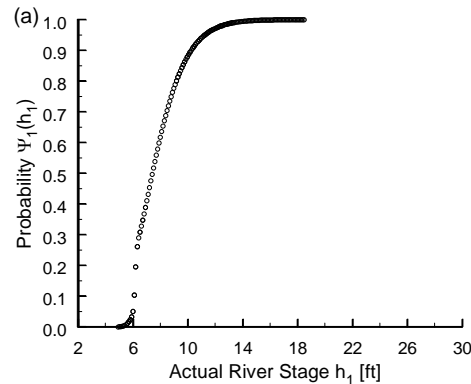
$$\bar{F}_n(h) = P(Z_n > h) \quad n = 1, \dots, N$$

$$Z_n = \max \{H_1, \dots, H_n\}$$

PRSF: Predictive Distribution of H_n

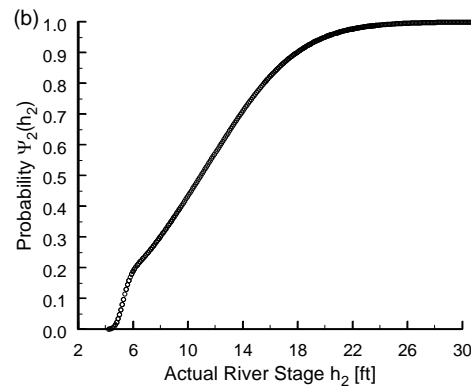
$n = 1$

lead time 24 h



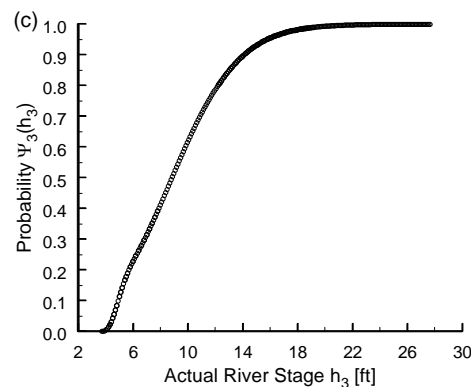
$n = 2$

lead time 48 h



$n = 3$

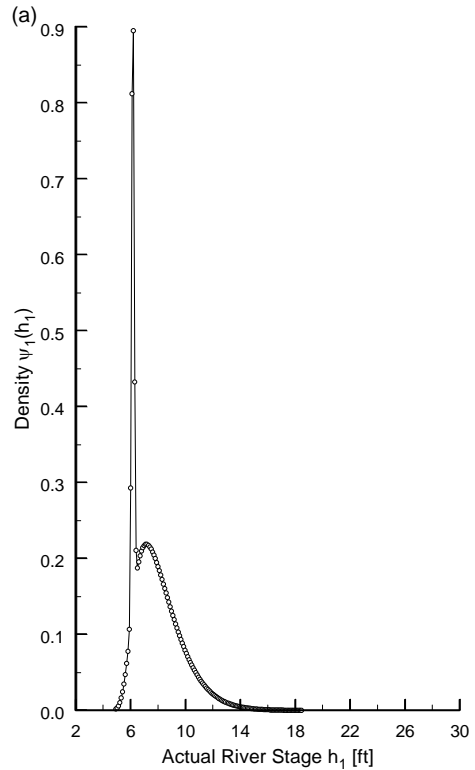
lead time 72 h



PRSF: Predictive Density of H_n

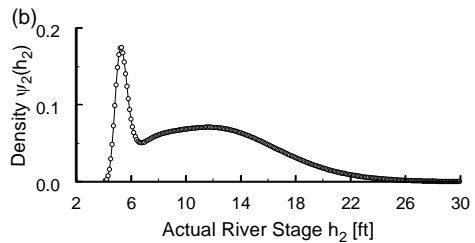
$$n = 1$$

lead time 24 h



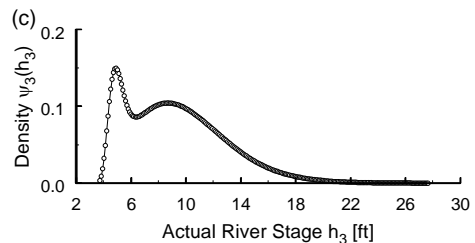
$$n = 2$$

lead time 48 h

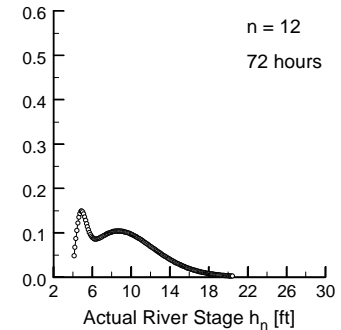
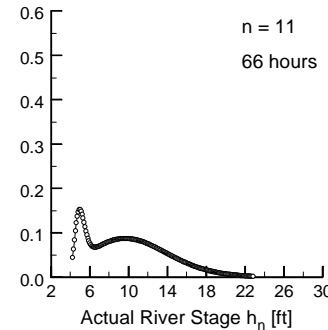
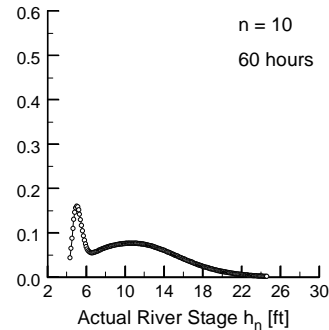
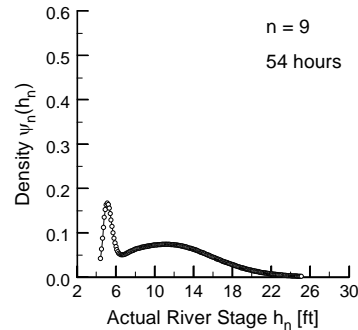
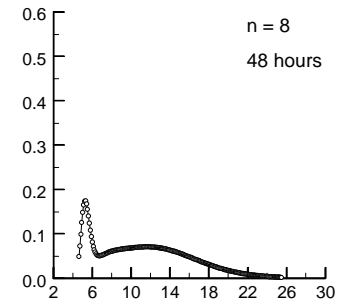
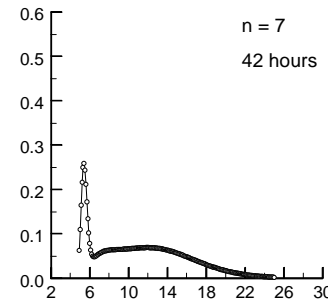
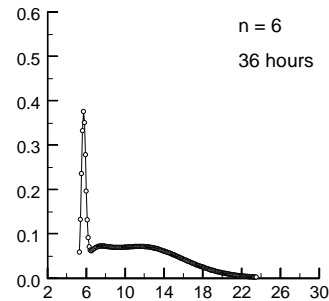
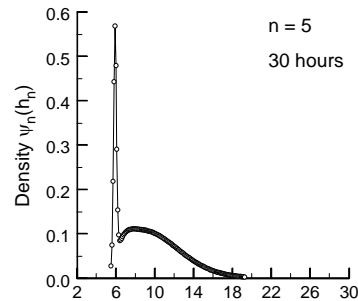
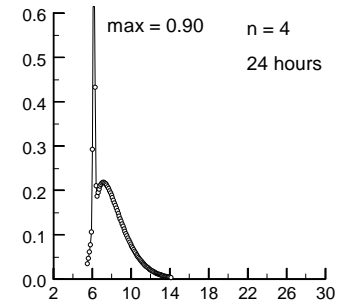
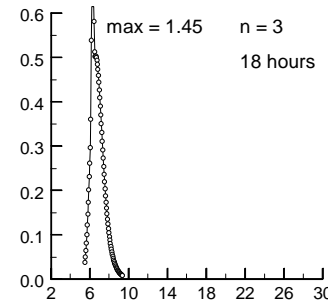
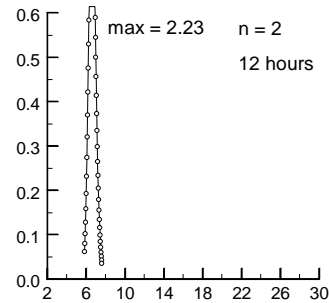
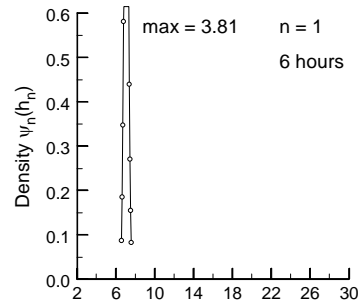


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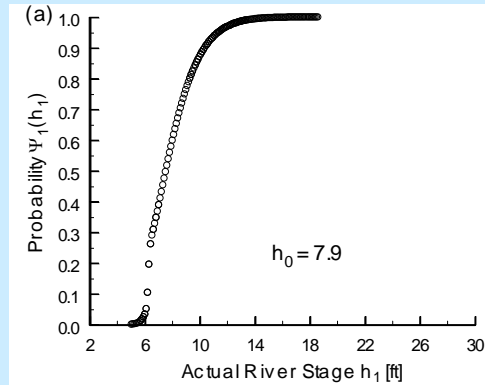
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PSTF: Predictive 1-Step Markov Transition

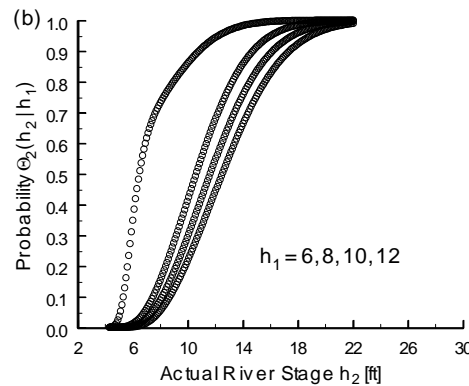
$n = 1$

lead time 24 h



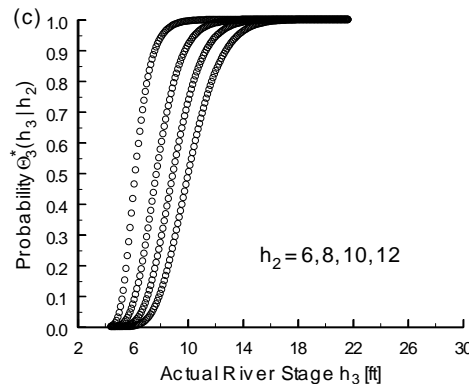
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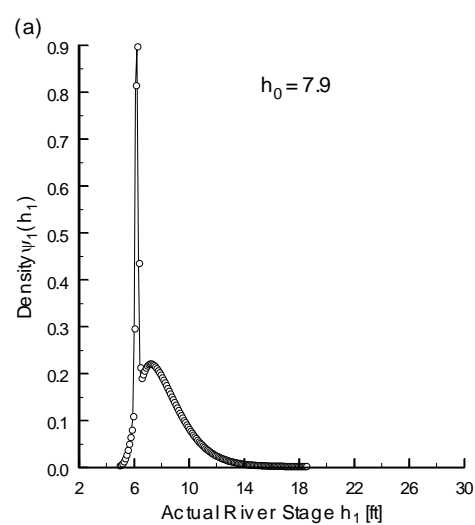
lead time 72 h



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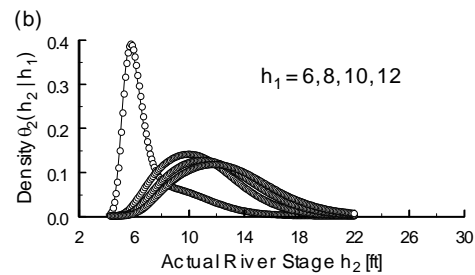
$$n = 1$$

lead time 24 h



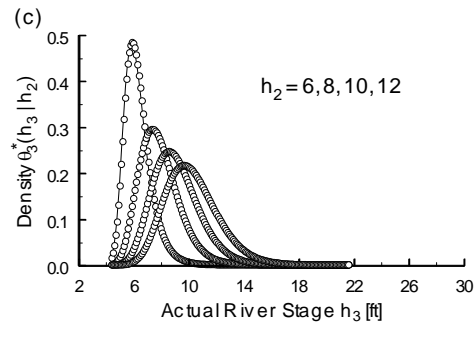
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lead time 48 h

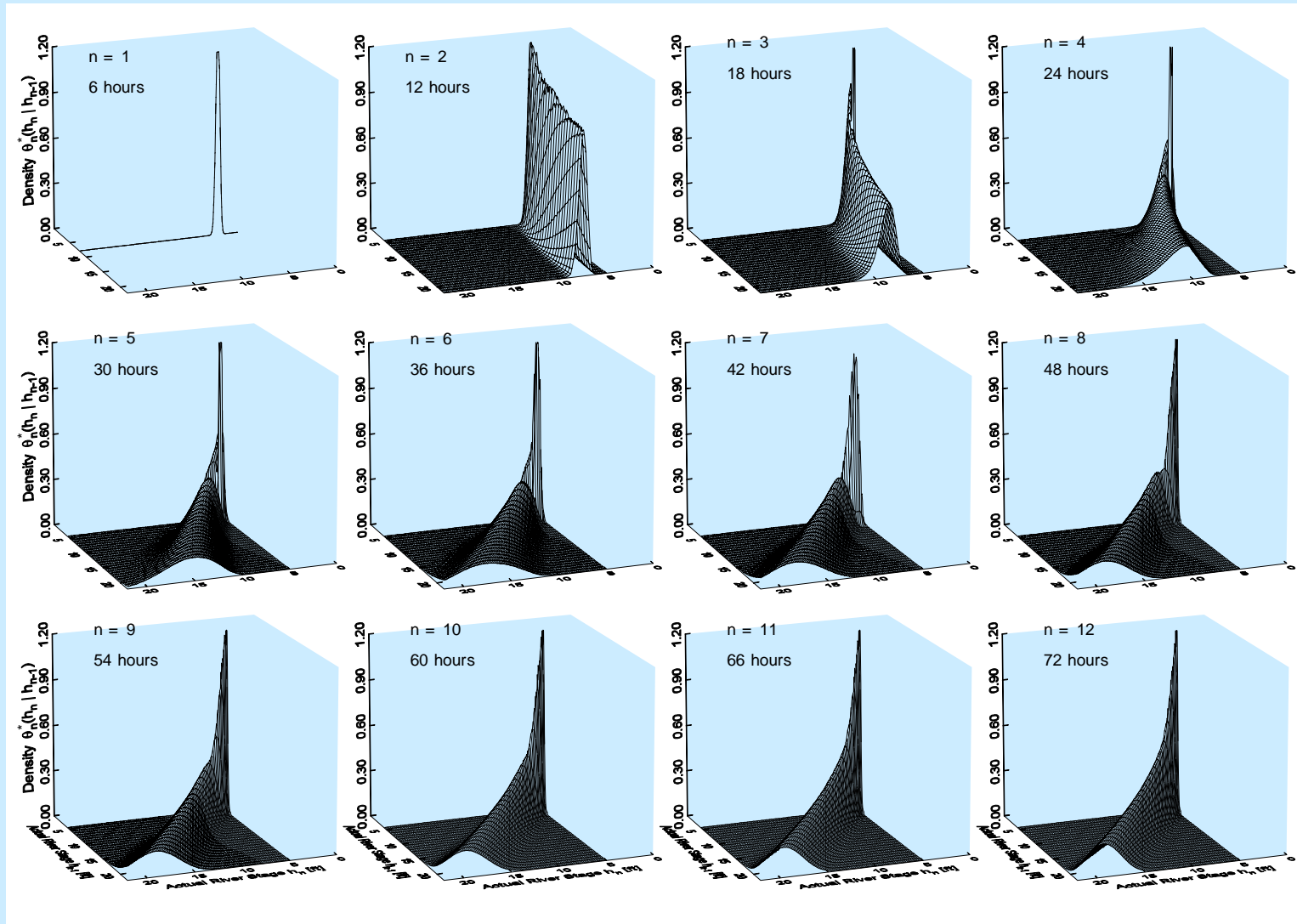


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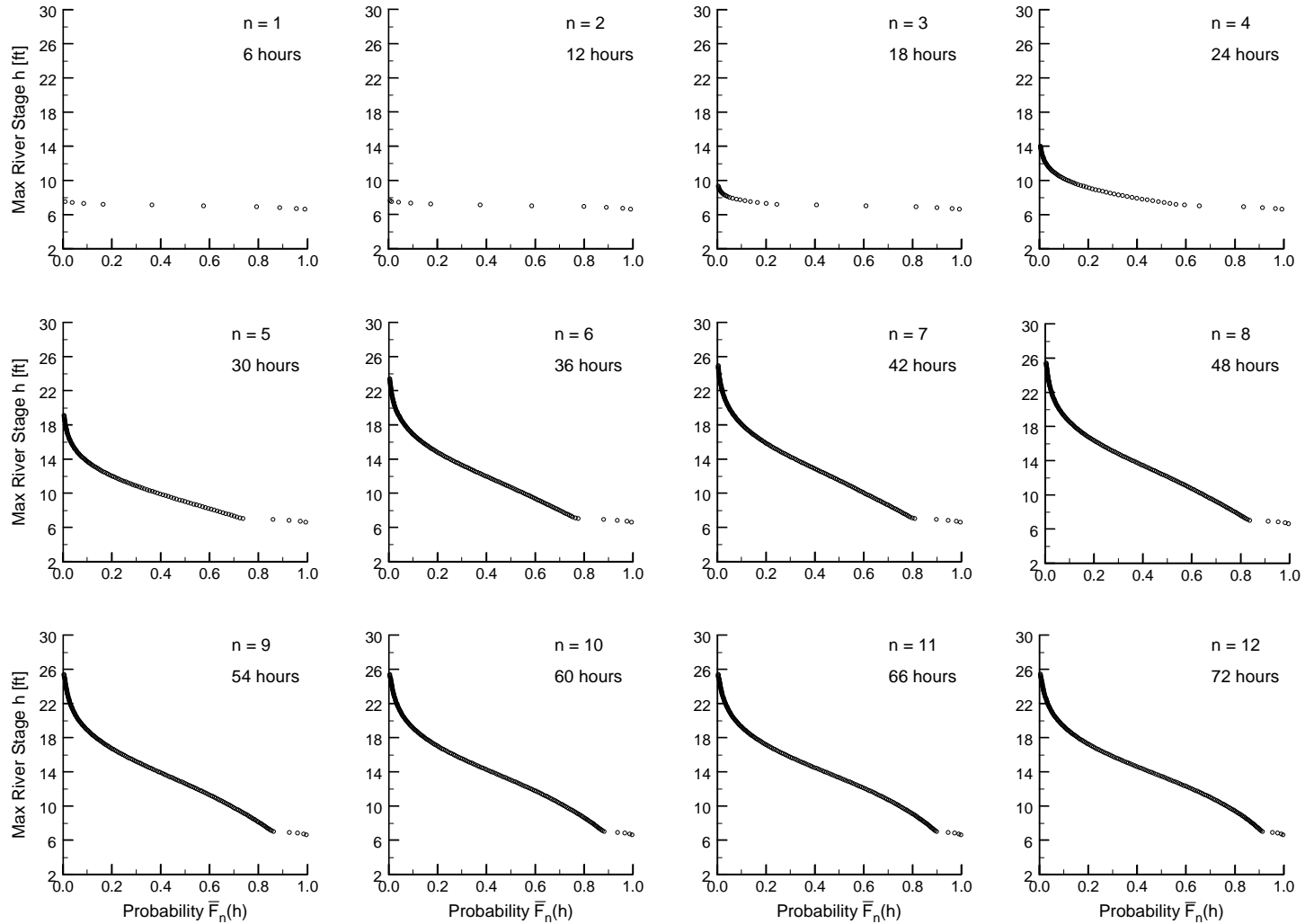
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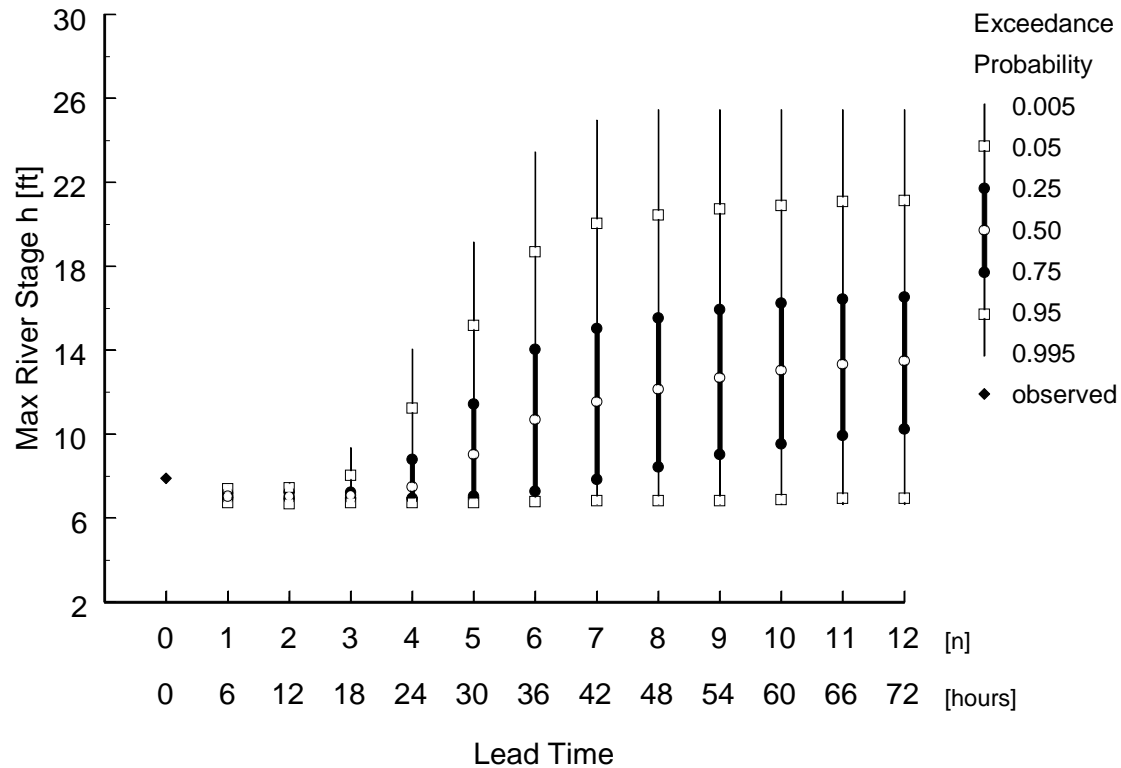
PFF: Exceedance of Max River Stage

$$\bar{F}_n(h) = P(Z_n > h)$$

$$Z_n = \max \{H_1, \dots, H_n\}$$

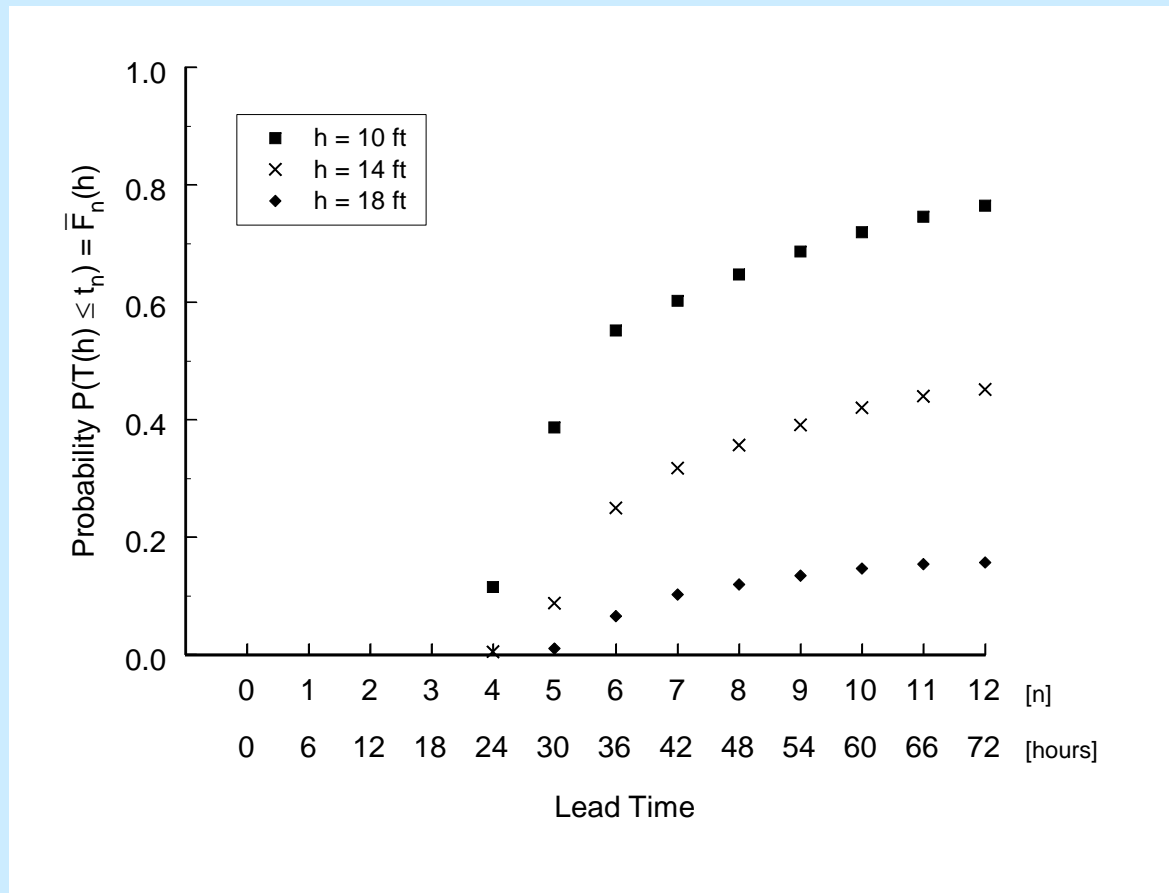


PFF: Isoprobability Time Series of Quantiles (and Credible Intervals) of Max River Stage



PFF: Distribution Function of Time to Flooding

$$P(T(h) \leq t_n) = \bar{F}_n(h) \quad n = 1, \dots, 12$$



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