BAYESIAN PROBABILISTIC QUANTITATIVE PRECIPITATION FORECASTS

By

Coire J. Maranzano and Roman Krzysztofowicz

University of Virginia

Poster Presentation at 18th Conference on Probability and Statistics in the Atmospheric Sciences 86th Annual Meeting of the American Meteorological Society Atlanta, Georgia, 29 January – 2 February 2006

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Corresponding author: Professor Roman Krzysztofowicz University of Virginia P.O. Box 400747 Charlottesville, VA 22904-4747 Tel: 434-982-2067, Fax: 434-982-2972 Email: rk@virginia.edu

Acknowledgments: Work supported by the National Science Foundation under Grant No. ATM-0135940.

Data provided by the Meteorological Development Laboratory of the National Weather Service.
Bayesian Processor of Output (BPO) for Probabilistic Quantitative Precipitation Forecasting

**APPRAOCH**

- Theoretically based technique
- Processes Numerical Weather Prediction (NWP)
- Optimally fuses climatic data with NWP model outputs
- Quantifies the uncertainty about weather variates
  - Binary predictands
  - Multi-category predictands
  - Continuous predictands

**PRODUCTS**

- \( W \) — precipitation amount (binary-continuous) \( W \geq 0 \)
- \( X \) — vector of predictors \( X = (X_1, \ldots, X_I) \)

PoP: Probability of Precipitation Occurrence

\[
\pi = P(W > 0 | X = x)
\]

DoA: Distribution of Precipitation Amount, Conditional on Occurrence

\[
\Phi(w) = P(W \leq w | X = x, W > 0), \quad w \geq 0
\]

PQPF: Probabilistic Quantitative Precipitation Forecast

\[
P(W \leq w | X = x) = (1 - \pi) + \pi \Phi(w), \quad w \geq 0
\]

**THEORY**

<table>
<thead>
<tr>
<th>Density Function</th>
<th>Model</th>
<th>Forecasting Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(w) )</td>
<td>prior (input)</td>
<td>univariate of any form</td>
</tr>
<tr>
<td>( f(x</td>
<td>w) = p(x</td>
<td>W = w) )</td>
</tr>
<tr>
<td>( \kappa(x) )</td>
<td>expected (output)</td>
<td>( l )-variate meta-Gaussian</td>
</tr>
<tr>
<td>( \phi(w) = p(w</td>
<td>X = x) )</td>
<td>posterior (output)</td>
</tr>
</tbody>
</table>

\[
\kappa(x) = \int_{-\infty}^{\infty} f(x|w) g(w) \, dw
\]

\[
\phi(w) = \frac{f(x|w)}{\kappa(x)} g(w)
\]

**ADVANTAGES**

- The PQPF
  - continuous distribution function for \( W > 0 \)
  - guaranteed to be well calibrated against the prior (climatic) distribution function
- The BPO
  - has correct theoretic structure
  - is more parsimonious than MOS
  - is easily extended to ensemble forecasting
Example: Distribution of Amount (DoA)

Station: Quillayute, WA  
Season: Cool  
Lead time: 12 h after 0000 UTC  
Date: 21 February 2002  
Date: 21 February 2002  
Forecast Period: 24 h  
Actual Precip. Amount: 101.85 mm

**Predictors**

BPO: 3 predictors; 15 parameters  
24-H TOTAL PRECIP. ending 36 h \(x_1 = 30.2\)  
850 REL. VORTICITY at 24 h \(x_2 = 4.8\)  
700 VERTICAL VELOCITY at 12 h \(x_3 = -0.95\)

**Forecast**

BPO: function  
- continuous  
- meta-Gaussian  
- closed-form

**MOS:** 15 predictors; 80 parameters (5 catego.)

12-H TOTAL PRECIP. GB (6.35 mm) ending 24 h  
12-H TOTAL PRECIP. GB (25.4 mm) ending 24 h  
12-H TOTAL PRECIP. GB (0.254 mm) ending 36 h  
24-H TOTAL PRECIP. GB (2.54 mm) ending 24 h  
24-H CONV. PRECIP. GB (0.254 mm) ending 36 h  
850 REL. VORTICITY at 12 h  
850 REL. VORTICITY at 24 h  
700 VERTICAL VELOCITY GB (-0.9) at 24 h  
700 VERTICAL VELOCITY GB (-0.5) at 12 h  
LATITUDE, LONGITUDE, ELEVATION

**Bayesian Revision**

**Distribution Functions**

Prior (climatic)

NWP Model Output  
\(W_{\text{Weibull}}(\alpha = 0.52, \beta = 0.88)\)

Posterior

\(x = (30.2, 4.8, -0.95)\)

**Density Functions**

Prior (climatic)

NWP Model Output  
\(W_{\text{meta-Gaussian}}(\alpha = 0.52, \beta = 0.88)\)

Posterior

\(x = (30.2, 4.8, -0.95)\)

**Hypothetical Forecasts of an Extreme**

Actual \(x_1 = 30.2\)

Hypothetical \(x_1 = 60.0\)

Hypothetical \(x_1 = 90.0\)

Hypothetical \(x_1 = 120.0\)
Example: Distribution of Amount (DoA)

Station: Buffalo, NY  
Season: Cool  
Lead time: 36 h after 0000 UTC  
Date: 30 January 2002  
Forecast Period: 6 h  
Actual Precip. Amount: 21.84 mm

**Predictors**

BPO: 2 predictors; 10 parameters  
- 6-H TOTAL PRECIP, ending 42 h \(x_1 = 11.2\)  
- 500 VERTICAL VELOCITY at 36 h \(x_2 = -0.55\)

MOS: 13 predictors; 42 parameters (3 catego.)  
- 6-H TOTAL PRECIP, GB (2.54 mm) ending 42 h  
- 6-H TOTAL PRECIP, GB (6.35 mm) ending 42 h  
- 500 VERTICAL VELOCITY GB (-0.9) at 42 h  
- 6-H PRECIP, WATER ending 42 h  
- 925 VERTICAL VELOCITY GB (-0.2) at 33 h  
- 3-H TOTAL PRECIP, GB (6.35 mm) ending 39 h  
- 850 VERTICAL VELOCITY GB (-0.5) at 45 h  
- 700 VERTICAL VELOCITY GB (-0.9) at 36 h  
- 700 VERTICAL VELOCITY GB (-1.2) at 36 h  
- 925 VERTICAL VELOCITY GB (-0.1) at 39 h  
- 500 VERTICAL VELOCITY GB (-0.5) at 42 h  
- 500 VERTICAL VELOCITY GB (-1.2) at 39 h  
- ELEVATION

**Forecast**

BPO: function  
- continuous  
- meta-Gaussian  
- closed-form

MOS: 3 points  
- Ad-hoc extrapolation needed

**Bayesian Revision**

**Distribution Functions**

Prior (climatic)  
NWP Model Output \(x = (11.2, -0.55)\)

**Density Functions**

Posterior

Prior (climatic)

NWP Model Output \(x = (11.2, -0.55)\)

**Uncertainty Quantification**

AVN model: Grossly underestimates the precipitation amount

MOS system: Categories inadequate to characterize uncertainty  
- Ad-hoc extrapolation needed

BPO system: Complete characterization of uncertainty — continuous DoA  
- Guaranteed calibration against prior distribution  
- Simpler predictors: direct NWP model output (no Grid-Binary)  
- Fewer predictors and parameters