Radiometric Dating and Half Life

Over time a radioactive sample becomes less radioactive as unstable atoms decay to stable forms.

The decay is a random event for any atom resulting in ½ of the remaining radioactive atoms decaying in a fixed time interval.

Each radioactive isotope has its own characteristic “half-life”

- $^{235}$U → $^{207}$Pb (704 million years)
- $^{87}$Rb → $^{87}$Sr (48.8 billion years)
- $^{14}$C → $^{14}$N (5730 years)

½ will remain after one half life
¼ will remain after two half lives
1/8 will remain after three half lives.

$$N(t) = N_0 \exp\left(-\ln 2 \frac{t}{t_{\text{half}}}\right)$$
Natural Radioactivity

Measuring the relative amounts of radioactive elements and their isotopes in a rock reveals the time since solidification.

Radioactivity is prevalent in everyday life

Stars forged virtually all of the atomic nuclei in the Earth.

Exploding stars produce vast quantities of radioactive debris.
Natural Radioactivity

The Earth formed from stellar debris and incorporated a small portion of radioactive material.

The heat generated by the decay of this material still warms the Earth's interior to temperatures high enough for volcanic activity.
Short and Long Lived Isotopes

• Radioactive isotopes were incorporated at the time of formation of the planets.
  – Some evidence points to the injection of “fresh” radioactivity just prior to the formation of the Solar System – a nearby supernova?

• Isotopes with short (millions of years or less) half lives have decayed entirely by now.
  – Their extinct isotopic signatures illuminate events early in the history of the Solar System/Earth.

• Long lived isotopes of uranium, thorium, and potassium with half lives around a billion years or more “power” the Earth's interior.

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<tbody>
<tr>
<td>$^{238}\text{U}$</td>
<td>$9.46 \times 10^{-5}$</td>
<td>$4.47 \times 10^9$</td>
<td>$30.8 \times 10^{-9}$</td>
<td>$2.91 \times 10^{-12}$</td>
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<td>$^{235}\text{U}$</td>
<td>$5.69 \times 10^{-4}$</td>
<td>$7.04 \times 10^8$</td>
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<td>$^{232}\text{Th}$</td>
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<td>$1.40 \times 10^{10}$</td>
<td>$124 \times 10^{-9}$</td>
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<td>$^{40}\text{K}$</td>
<td>$2.92 \times 10^{-5}$</td>
<td>$1.25 \times 10^9$</td>
<td>$36.9 \times 10^{-9}$</td>
<td>$1.08 \times 10^{-12}$</td>
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Radiometric Dating and Half Life

Over time a radioactive sample becomes less radioactive as unstable atoms decay to stable forms.

The decay is a random event for any atom resulting in $\frac{1}{2}$ of the remaining radioactive atoms decaying in a fixed time interval.

Each radioactive isotope has its own characteristic “half-life”

- $^{235}\text{U} \rightarrow ^{207}\text{Pb}$ (704 million years)
- $^{87}\text{Rb} \rightarrow ^{87}\text{Sr}$ (48.8 billion years)
- $^{14}\text{C} \rightarrow ^{14}\text{N}$ (5730 years)

$\frac{1}{2}$ will remain after one half life

$\frac{1}{4}$ will remain after two half lives

$\frac{1}{8}$ will remain after three half lives.
Exponential Decay

- Why is radioactive decay characterizable by “half-lives”
  - Any multiplicative (halving or doubling) process is an exponential process.
  - For radioactive decay, the exponential process arises because there is a fixed probability that any given particle will decay in any given time interval. The number of decays per unit time is simply proportional to the number of particles.

\[
\frac{dN}{dt} = -\lambda N(t)
\]

- The equation above is the differential form of an exponential law with solution.

\[
N(t) = e^{-\lambda t + c} = N_0 e^{-\lambda t}
\]
Population Growth

- Population growth follows the same model, but in the opposite direction. The number of new humans per year is proportional to the number of existing humans.
  - In this problem there is also a “decay” exponential due to death.
  - Unfortunately the birthrate factor exceeds the deathrate factor and modern medicine is making the gap larger.

\[
\frac{dN}{dt} = (\lambda_B - \lambda_D) N(t)
\]
Radioactive Decay

• The decay constant, $\lambda$, maps to half life simply by requiring

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}} \quad T_{1/2} = \frac{\ln(2)}{\lambda}$$

• The number of daughter atoms at time, $t$, is $N_o - N(t)$

$$N_D = N_o (1 - e^{-\lambda t}) = N(t)(e^{\lambda t} - 1)$$

• In simple terms then

$$t = \ln\left(1 + \frac{N_D}{N}\right)/\lambda$$
Radiometric Dating: Slightly More Complicated than Just Half Lives

How do you account for the amount of decay product in the rock on day one?

Elements fractionate. Isotopes don't.

At time zero all rock crystals have the same ratio of $^{87}\text{Sr}$ to $^{86}\text{Sr}$, but different ratios of $^{87}\text{Rb}$ to $^{86}\text{Sr}$.

Melting an “aged” rock (sloped isochron) mixes the grain content and re-levels the isotope ratio for Sr.

$$N(t) = N_0 \exp\left(-\ln 2 \frac{t}{t_0}\right)$$
Interpretation of the Isochron Slope

Writing a simple statement that the present number of strontium atoms is the original number plus the rubidiums that have decayed...

\[
^{87}\text{Sr}(t) = ^{87}\text{Sr}_0 + \left(^{87}\text{Rb}_0 - ^{87}\text{Rb}(t)\right)
\]

\[
^{87}\text{Rb}(t) = ^{87}\text{Rb}_0 \exp\left(-\ln 2 \frac{t}{t_{\text{half}}}\right)
\]

\[
^{87}\text{Rb}_0 = ^{87}\text{Rb}(t) \exp\left(\ln 2 \frac{t}{t_{\text{half}}}\right)
\]

\[
^{87}\text{Sr}(t) = ^{87}\text{Sr}_0 + ^{87}\text{Rb}(t)\left(\exp\left(\ln 2 \frac{t}{t_{\text{half}}}\right) - 1\right)
\]

\[
\frac{^{87}\text{Sr}(t)}{^{86}\text{Sr}} = \frac{^{87}\text{Sr}_0}{^{86}\text{Sr}} + \frac{^{87}\text{Rb}(t)}{^{86}\text{Sr}}\left(\exp\left(\ln 2 \frac{t}{t_{\text{half}}}\right) - 1\right)
\]
The Oldest Rocks in the Solar System

- Melting resets the isochron clock by mixing grains that have evolved significantly different ratios of isotopes of the same element through radioactive decay.
- Earth rocks are relatively young having melted and re-solidified mostly in the last billion years.
- Asteroids have been largely un-melted since the formation of the Solar System.
Terrific Uniformity in Meteorite Radiometric Ages

- Murchison (CM 2.5)
- Y791198 (CM 2.4)
- ALH83100 (CM 2.1)
- ALH84051 (CM 2.1)
- ALH84034 (CM 2.1)
- QUE93005 (CM 2.1)
- Sayama (CM 2.1)
- LEW86010 (angrite)

Legend:
- □ Previous studies
- ■ This study

Age (Myr ago)
Radioactive Dating Applied to Impact Melted Spherules in Lunar Soil
Calcium/Aluminum Rich Inclusions (CAI)

- These highly refractory grains form at the highest temperatures and earliest times in the Solar Nebula.
- Radiometric ages of about a million years older than the rest of the meteorite matrix (chondrules) are consistent with this scenario.
Ca-Al Rich Inclusions (CAI)

- These inclusions show an excess of Magnesium-26 which is the decay product of Aluminum-26.
  - Aluminum-26 decays energetically with a half life of 0.7 million years
    - Suggests injection just prior to solar system formation (a supernova trigger?)
    - Provides more than enough energy to melt small bodies – differentiated asteroids.
Calcium/Aluminum Rich Inclusions (CAI)
Extinct Radioactivities

- Short-lived radionuclides have all decayed away, but their isotopic signatures can illuminate processes happening in the earliest stages of Solar System development during the first several half lives.
  - Aluminum-26/Magnesium-26 (0.7 million years)
  - Hafnium-182/Tungsten-182 (9 million years)
  - Hafnium (Hf) is a lithophile element while Tungsten (W) is a siderophile

**Figure 3. Abundances of elements** in Earth’s silicate crust and mantle, normalized to abundance in CI chondrite meteorites, are plotted against each element’s volatility, as measured by the condensation temperature \( T_c \) at which 50% of its atoms in the protoplanetary gas disk would have condensed. Elements with high \( T_c \) and therefore low volatility are called refractory. Their readiness to dissolve in molten iron is color-coded as siderophile (iron loving) or lithophile (rock loving). The figure shows increasing depletion in Earth’s crust and mantle (relative to the meteorites) with increasing volatility and siderophilicity. (Adapted from ref. 6.)

Physics Today Article on Timing of Formation of Earth’s Core based on Hafnium/Tungsten isotope ratios.
Formation of the Earth's Core

- Key fact, the Earth likely formed over the course of about 100 million years following roughly an exponential law.

_Earth’s first 100 million years._ Earth grew mainly from a series of accretion events (15) broadly comparable in scale to the giant impact, which formed the Moon ~100 million years after the start of the solar system (8, 10). In disequilibrium accretion (A), the giant impact added metal directly to the core. In equilibrium accretion (B), metal segregated from silicate in a magma ocean. The concentrations of metal-loving elements left in the silicate Earth after core formation are better explained by scenario (B) (12). However, W isotope data then imply that accretion and core formation were mainly rapid, possibly with a substantial hiatus before the giant impact.
Formation of the Earth's Core

- Infalling material was of solar composition as illuminated by CI chondritic meteorite composition.
Formation of the Earth's Core

- Material was “cooked” at great pressure in a deep magma ocean precipitating siderophiles onto the core, including Tungsten.

Differentiation

Figure 4. Deep magma ocean model of core formation. The high pressures required by partitioning of siderophile elements from Earth’s mantle to its core imply that iron descended in droplets through a deep molten silicate layer, equilibrating with the silicates as they fell. Because the pressures are much lower than at the core–mantle boundary, it’s conjectured that the molten iron and siderophile elements dissolved in it formed a pond shell at some intermediate depth where there was a transition from molten to solid silicate. The dense metal-pond layer eventually became gravitationally unstable so that large metal blobs descended to the core. (Adapted from ref. 6.)
Formation of the Earth's Core

- Infall continues resupplying fresh Hafnium/Tungsten to a Hafnium enriched/Tungsten depleted surface layer.
  - Applying this technique in different environments give a core formation time of 2 Myr in asteroids, 7 Myr for Mars, and 30 Myr for Earth.

**Figure 5. The isotopic ratio** of two stable tungsten isotopes, $^{182}\text{W}$ and $^{184}\text{W}$, measured in different systems, determines the time at which a planet's core separated from its silicate mantle—assuming core formation was abrupt. Tungsten-182 is produced by the (9-Myr half-life) decay of the now extinct hafnium-182. Because Hf is lithophile, an iron core forming in a body of chondritic Hf/W abundance has its W isotopic ratio frozen in time because the core contains no Hf. By contrast, the silicate mantle goes on to evolve a W isotopic ratio higher than that in chondrites. The horizontal axis marks time since the solar system's beginning. After about 50 Myr, the W isotopic ratio no longer grows in the mantle or the chondrites because there's no more $^{182}\text{Hf}$ left.
Crater Counting Chronologies

- Planetary radius determines the extent to which radioactive decay can maintain geological activity.
Crater Counts vs. Size vs. “Age”

- Craters accumulate over time on a “fresh” surface.
- Eventually the crater density saturates.
- Large impacts can also produce large numbers of secondary craters.