Flux vs. Flux Density

- Flux is the amount of energy from a source that passes through a square centimeter a distance, d, away integrated over all wavelengths.
  - Also referred to as “bolometric flux”. Units are erg/cm²/s or watts/m²

\[ \text{Flux} = \frac{\text{Luminosity}}{4\pi d^2} \]

- Flux density accounts for the fact that the energy passing through that square centimeter is wavelength dependent and we may care, for example, about the energy traversing that square centimeter in a particular filter bandpass.
  - Units are erg/cm²/s/µm or watts/m²/Hz
Integrating Flux Across a Filter Bandpass

• Measuring a flux requires looking through a filter at an astronomical source.
  – That filter has a bandpass (variable transmission with wavelength)
  – The target has a spectrum (structure within the filter bandpass)
Integrating Flux Across a Filter Bandpass

\[ \text{filter\_flux}_{\text{Band}} = \int_0^\infty \text{filter\_transmission}(\lambda) \ast \text{target\_flux\_density}(\lambda) \, d\lambda \]
Integrating Flux Across a Filter Bandpass

\[ \text{filter}_\text{flux}_{\text{Band}} = \int_0^\infty \text{filter}_\text{transmission}(\lambda) \times \text{target}_\text{flux}_\text{density}(\lambda) \, d\lambda \]
Integrating Flux Across a Filter Bandpass

- To be strictly correct one must account for atmospheric and telescope/instrument transmission as a function of wavelength.

\[
\text{filter\_flux}_{Band} = \int_{0}^{\infty} \text{filter\_transmission}(\lambda) \ast \text{target\_flux\_density}(\lambda) \ast \text{atm\_trans}(\lambda) \ast \text{inst\_trans}(\lambda) \, d\lambda
\]

- Measurement in counts = \(\text{filter\_flux} \ast \text{QE} \ast \text{gain} = \text{UNCALIBRATED}\)

- Observing two stars simultaneously (in the same image for example) calibrates out the time variable terms, \(\text{atm\_trans}\) in particular.
Magnitudes for Real

- From ASTR2110, 2120, etc... consider two targets with measured fluxes $f_1$ and $f_2$

$$m_1 - m_2 = -2.5 \log \left( \frac{f_1}{f_2} \right)$$

- A flux ratio of 10 is a difference of 2.5 magnitudes, a flux ratio of 100 is 5 magnitudes.
Instrumental Magnitudes

- Since magnitudes are ratios, any constant of proportionality that scales the measurement (for example converting instrumental counts to physical flux units) cancels out.

\[ m_1 - m_2 = -2.5 \log\left(\frac{\phi f_1}{\phi f_2}\right) \]

- The simple implication is that you can evaluate magnitude differences simply by applying the magnitude equation to extracted instrumental counts.

\[ m_{\text{instrumental}} = -2.5 \log\left(\text{counts}\right) \]
Magnitudes, Percentages, and SNR

• Consider the flux difference of two stars that differ by 0.1 magnitude
  – Since magnitudes are logarithmic a magnitude difference corresponds to a
    multiplicative/factor difference in brightness so it doesn’t matter if we are
    talking about magnitude 10.0 vs. 10.1 or 4.0 vs. 4.1

• It turns out an 0.1 mag difference is close to a 10% difference in flux
  and an 0.01 mag difference is close to a 1% difference because....

• Consider an 0.1 mag difference

\[ m_1 - m_2 = 0.1 = -2.5 \log \left( \frac{f_1}{f_2} \right) \]
\[ \frac{f_1}{f_2} = 10^{-\left(\frac{0.1}{2.5}\right)} \]

• Taylor series expand this small difference: \( f(x + \Delta x) = f(x) + f'(x) \Delta x + ... \)

• Expanding about \( x=0 \)

\[ 10^x = 1 + \ln(10) \Delta x \quad \text{for} \quad x = -0.1 / 2.5 \quad 10^x = 1 - 2.3 \times \left( \frac{0.1}{2.5} \right) \]
Magnitudes, Percentages, and SNR

\[ \frac{f_1}{f_2} = 10^{-\left(\frac{0.1}{2.5}\right)} \]

- Expanding around \( x=0 \)

\[ 10^x = 1 + \ln(10) \cdot \Delta x \quad \text{for} \quad x = -\left(\frac{0.1}{2.5}\right) \quad 10^x = 1 - 2.3 \cdot \left(\frac{0.1}{2.5}\right) = 0.91 \]

- So, thanks to the fact that \( \ln(10) \) is pretty close to 2.5 the flux ratio for 0.1 mag difference corresponds to about a 10% difference in flux.

- Similarly, if a magnitude measurement is uncertain by +/- 0.1 mag that uncertainty translates to 10% or SNR=10.
  - 10.73 +/- 0.01 mag is about a 1% uncertainty in magnitude corresponding to SNR=100
Photometry is Ultimately “Comparative” (even though it strives to be absolute!)

• The magnitude equation always quantifies a difference (which is actually a flux ratio) and it is “easy” to measure ratios.
• As physicists, however, we would like to know the answers in SI units.
  – Fluxes in erg/cm²/s or flux densities in erg/cm²/s/µm...
• In rare instances an instrument calibrated to SI units gets pointed at a star.
  – For the most part we are left to measure magnitude differences between stars (via instrumental counts – which are proportional to fluxes) and bootstrap our way to a physically calibrated standard.
  – Modern sky surveys provide billions of calibrated sources at a variety of wavelengths (there might always be a calibrator in your field of view).
  – Image two stars at the same time, extract the counts, and you get the flux ratio/magnitude difference independent of the atmosphere (to first order).

\[ m_1 - m_2 = -2.5 \log \left( \frac{f_1}{f_2} \right) \]
Magnitude Zeropoints

\[ m_{calibrated} = -2.5 \log \left( \frac{f_{\text{star}}}{f_{zp}} \right) \]

\[ = -2.5 \log(f_{\text{star}}) + 2.5 \log(f_{zp}) \]

zero point expressed as flux, or
flux density at the nominal
wavelength of the filter

“zero point” because a (special, agreed upon) star with flux=flux_zp will
have mag=?
Magnitude Zeropoints

\[ m_{\text{calibrated}} = -2.5 \log \left( \frac{f_{\text{star}}}{f_{zp}} \right) \]

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zero point expressed as flux, or flux density at the nominal wavelength of the filter

“zero point” because a (special, agreed upon) star with flux=flux_zp will have mag=?

\[ = -2.5 \log(\text{cts}_{\text{star}}) + 2.5 \log(\text{cts}_{zp}) \]

\[ = -2.5 \log(\text{cts}_{\text{star}}) - m_{zp} \]

“zero point magnitude” used to calibrate counts e.g. archival HST data

/ PHOTOMETRY KEYWORDS
COMMENT 1 blank line
PHOTMODE = 'WFC3 IR F160W' / observation con
PHOTFLAM = 1.9275602E-20 / inverse sensitivity, ergs/cm2/Ang/revolution
PHOTPOLE = 1.3167370E-07 / inverse sensitivity, Jy sec/revolution
PHOTZPT = -2.1100000E+01 / ST magnitude zero point
PHOTPLAM = 1.5369176E+04 / Pivot wavelength (Angstroms)
PHOTBW = 8.2625085E+02 / RMS bandwidth of filter plus detector
Magnitude Zeropoints

- $0^{\text{th}}$ magnitude requires a definition (a comparison poster-child).
- Since measuring relative fluxes is “easy” the star Vega is defined to be magnitude 0.00 in all bands.

$$m_{\text{star}} = -2.5 \log\left(\frac{f_{\text{star}}}{f_{\text{Vega}}}\right)$$

- At some point Vega has been quantitatively assessed in every filter

### Vega Flux Zeropoints

<table>
<thead>
<tr>
<th>Quantity</th>
<th>U</th>
<th>B</th>
<th>V</th>
<th>R</th>
<th>I</th>
<th>J</th>
<th>H</th>
<th>K</th>
<th>Notes and units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{eff}}$</td>
<td>0.36</td>
<td>0.438</td>
<td>0.545</td>
<td>0.641</td>
<td>0.798</td>
<td>1.22</td>
<td>1.63</td>
<td>2.19</td>
<td>microns</td>
</tr>
<tr>
<td>$\Delta\lambda$</td>
<td>0.06</td>
<td>0.09</td>
<td>0.085</td>
<td>0.15</td>
<td>0.15</td>
<td>0.26</td>
<td>0.29</td>
<td>0.41</td>
<td>microns, UBVRI from Bessell (1990), JHK from AQ</td>
</tr>
<tr>
<td>$f_\nu$</td>
<td>1.79</td>
<td>4.063</td>
<td>3.636</td>
<td>3.064</td>
<td>2.416</td>
<td>1.589</td>
<td>1.021</td>
<td>0.64</td>
<td>$x10^{-20}$ erg cm$^{-2}$ s$^{-1}$ Hz$^{-1}$, from Bessell et al. (1998)</td>
</tr>
<tr>
<td>$f_\lambda$</td>
<td>417.5</td>
<td>632</td>
<td>363.1</td>
<td>217.7</td>
<td>112.6</td>
<td>31.47</td>
<td>11.38</td>
<td>3.961</td>
<td>$x10^{-11}$ erg cm$^{-2}$ s$^{-1}$ A$^{-1}$, from Bessell et al. (1998)</td>
</tr>
<tr>
<td>$\Phi_\lambda$</td>
<td>756.1</td>
<td>1392.6</td>
<td>995.5</td>
<td>702.0</td>
<td>452.0</td>
<td>193.1</td>
<td>93.3</td>
<td>43.6</td>
<td>photons cm$^{-2}$ s$^{-1}$ A$^{-1}$, calculated from above quantities</td>
</tr>
</tbody>
</table>
AB Magnitudes

- Just to make things more confusing, about 20 years ago a separate set of magnitude zero points were established to take the empirical measurement of Vega “out of the equation” and tie it to the fundamental units of flux density.

\[ m_{AB} = -2.5 \log f_v + 48.600 = -2.5 \log \left( \frac{f_v}{3631 \text{ Jy}} \right) \]

- \( f_v \) here is in units of erg/s/cm\(^2\)/Hertz
  - One “Jansky” is \(10^{-26}\) Watts/m\(^2\)/Hz

- The constant, 48.600, makes \( AB = V \) for a flat spectrum source

- [See http://adsabs.harvard.edu/doi/10.1086/160817](http://adsabs.harvard.edu/doi/10.1086/160817)
Registering AB and Vega Magnitudes

**Vega - AB Magnitude Conversion**

<table>
<thead>
<tr>
<th>Band</th>
<th>$\lambda_{\text{eff}}$</th>
<th>$m_{\text{AB}} - m_{\text{Vega}}$</th>
<th>$M_{\text{Sun}}(\text{AB})$</th>
<th>$M_{\text{Sun}}(\text{Vega})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>3571</td>
<td>0.79</td>
<td>6.35</td>
<td>5.55</td>
</tr>
<tr>
<td>B</td>
<td>4344</td>
<td>-0.09</td>
<td>5.36</td>
<td>5.45</td>
</tr>
<tr>
<td>V</td>
<td>5456</td>
<td>0.02</td>
<td>4.80</td>
<td>4.78</td>
</tr>
<tr>
<td>R</td>
<td>6442</td>
<td>0.21</td>
<td>4.61</td>
<td>4.41</td>
</tr>
<tr>
<td>I</td>
<td>7994</td>
<td>0.45</td>
<td>4.52</td>
<td>4.07</td>
</tr>
<tr>
<td>J</td>
<td>12355</td>
<td>0.91</td>
<td>4.56</td>
<td>3.65</td>
</tr>
<tr>
<td>H</td>
<td>16458</td>
<td>1.39</td>
<td>4.71</td>
<td>3.32</td>
</tr>
<tr>
<td>$K_s$</td>
<td>21603</td>
<td>1.85</td>
<td>5.14</td>
<td>3.29</td>
</tr>
<tr>
<td>u</td>
<td>3546</td>
<td>0.91</td>
<td>6.38</td>
<td>5.47</td>
</tr>
<tr>
<td>g</td>
<td>4670</td>
<td>-0.08</td>
<td>5.12</td>
<td>5.20</td>
</tr>
<tr>
<td>r</td>
<td>6156</td>
<td>0.16</td>
<td>4.64</td>
<td>4.49</td>
</tr>
<tr>
<td>i</td>
<td>7472</td>
<td>0.37</td>
<td>4.53</td>
<td>4.16</td>
</tr>
<tr>
<td>z</td>
<td>8917</td>
<td>0.54</td>
<td>4.51</td>
<td>3.97</td>
</tr>
<tr>
<td>Y</td>
<td>10305</td>
<td>0.634</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These data are mostly from Blanton et al. (2007).
Atmospheric Transmission vs. Wavelength

- Solution 1 – leave the atmosphere behind

**Images:**
- Hubble – Ultraviolet, Visible, Infrared
- Spitzer - Infrared
- CHANDRA X-ray Observatory
- Compton Gamma-ray Observatory
Atmospheric Transmission

- Molecular absorption, water in particular, contributes substantial atmospheric opacity in the infrared.
Mauna Kea – 14,000 feet
Atmospheric Transmission in the Submillimeter

- The Atacama Large Millimeter Array (ALMA) is sited at 17,000 feet altitude in one of the driest deserts on Earth.
Atmospheric Transmission

• Since water predominately resides in the troposphere, you just have to get into the stratosphere to see into space.
Atmospheric Transmission

• Since water predominately resides in the troposphere, you just have to get into the stratosphere to see into space.

SOFIA flies at 40,000 feet. High altitude balloons can take massive payloads to 120,000 feet.
Filter Bandpasses

- Calibrating observations precisely is dependent upon having precisely defined bandpasses.
Infrared Bandpasses

- Atmospheric absorption provides natural boundaries in wavelength for defining infrared filter bandpasses.
Stellar Photometry with Filters

- Differences between magnitudes (which are ratios when you think about it) measured in different filters are diagnostic of temperature of blackbodies (stars).
Stellar Photometry with Filters

- These color differences become more diagnostic (for example of luminosity class) when you account for stellar spectral features and how they change with stellar surface gravity.
Filter Characteristics/Terminology

- Cutoff and cut-on wavelengths
  - Defined by ½ power points at filter edges
- Center wavelength: ½(cutoff + cut-on)
- Bandpass: (cutoff – cut-on)
- Effective wavelength: Weighted average of product of transmission and wavelength (centroid wavelength accounting for transmission)

- Refer to on-board discussion
Hertz vs. Nanometers

• Converting a bandpass from wavelength units to Hertz is not as simple as dividing the wavelength into the speed of light.
  – One must differentiate the wavelength/frequency equation to get the proper conversion:

\[
\lambda \nu = c \quad \nu = \frac{c}{\lambda} \quad \Delta \nu = \frac{c}{\lambda^2} \Delta \lambda
\]
Filter Wheels
Filter Technologies

- Filters can simply be colored glass with the dye/glass transmission enforcing a transmission spectrum vs. wavelength.

- Alternatively filter bandpasses can be custom designed using multi-layer reflective(!) coatings.
  - The conspiracy of reflected and transmitted waves interfering with each other can produce a remarkably “square” high transmission bandpass.
Interference Filters vs. Angle of Incidence

- The transmission of bulk colored glass is independent of angle.
- Because interference filters rely on phase delay between multiply reflected waves, angles increase pathlengths and decrease(!) the effective wavelength of the filter.

![NF633-25 Transmission Graph](image-url)
Atmospheric Extinction

- Calibrating stellar photometry requires correction for loss of light passing through the atmosphere.
Time Variability of Extinction

• Extinction
  – Rayleigh scattering (optical; proportional to static pressure and airmass)
  – Ozone (optical)
  – Water (IR)
  – Volcanic aerosols
    • Can vary by 0.1-1%
    • Episodic problem
    • IR impact uncertain

Nabro eruption, 13 June 2011 (Bourassa, et al. (2012))
Extinction Correction in Practice

- In each filter measure the star at a variety of airmasses ($\Delta x$ below is $(\text{airmass} - 1)$) and determine the extinction in units of magnitudes per airmass for each observing band.