Consider a 6" BR7 objective with a 48" focal length (f/8)

\[
a) \text{Platescale} = \frac{1}{f.l. (\text{mm})} \times 206265 \frac{\text{arcsec}}{\text{mm}} = \frac{1}{48.0 \times 25.4 \text{mm}} \times 206265 = 170''/\text{mm}
\]

b) Compare f.l. at 500 nm vs. 400 nm

We know for the 500 nm refractive index the focal length is 48.0", exactly.

The lensmakers equation relates refractive index to focal length with a constant here:

\[\frac{1}{f} = (n-1) \left[ \text{something to do with curvatures} \right]
\]

\[n \text{ at 500 nm is } 1.5214 \text{ (400 nm is } 1.5310)\]

Let's start working in mm \(\Rightarrow 48.0'' = 1220 \text{ mm}\) (since the focus shift due to refraction is relative the exact number 1220 for 500 nm will do.)

\[\frac{1}{1220} = (1.5214-1) \Rightarrow C = 1.572 \times 10^{-3}\]

Knowing "C" we can now get the 400 nm f.l.

\[\frac{1}{x} = (1.5310-1) \times 1.572 \times 10^{-3} \Rightarrow x = 1197.9 \text{ mm}\]
c) So the 500nm focus is different from the 400 nm focus by \(1220 - 1197.9 \text{ mm} = 22.1 \text{ mm}\)

\[
\begin{array}{c}
\text{400 nm focus} \\
\text{500 nm focus}
\end{array}
\]

Since the beam expands at f/8 beyond the 400 nm focus (getting wider by 1 unit for every 8 units downstream), it will have broadened by \(\frac{22.1 \text{ mm}}{8} = 2.75 \text{ mm}\) (476") by the time it reaches the 500 nm focal point.

2) We want to have enough mirror actuators to compensate for seeing at 1250 nm. We know for 1" seeing at 500 nm \(R_0 = 10 \text{ cm}\).

The required number of actuators is the number of 1250 nm \(R_0\)'s that will "fit" in the telescope aperture, so

\[
N = \left(\frac{\text{Diam}}{R_0}\right)^2
\]

We must scale the 500nm, 1" seeing \(R_0 = 10 \text{ cm}\) to 1250nm and 0.8" seeing. Fortunately the course notes provide:

\[R_0 \propto \lambda^{1/2} \quad \text{and} \quad \text{seeing} \propto \frac{1}{\lambda R_0}\]

So for 0.8" seeing at 1250 nm \(R_0 = 10 \text{ cm} \times \frac{1}{0.8} = 12.5 \text{ cm}\)

The corresponding \(R_0\) at 500 nm \(= \frac{1250}{500}^{1.2} = 38 \text{ cm}\)

\[
N = \left(\frac{\text{Diam}}{R_0}\right)^2 = \left(\frac{650 \text{ cm}}{38 \text{ cm}}\right)^2 = 300 \text{ in round numbers}
\]