Spherical Aberration

- An “on-axis” aberration which arises from different radial zones on an optic producing a focus at different distances.

- By its geometrical definition, a parabola is free of spherical aberration (but guilty of others).

**Fig. 6K.** Spherical aberration of a concave spherical mirror.
Quantifying Aberrations

• The paraxial approximation applies for cases where rays intersect optics at small angle, \( \theta \), so that \( \sin \theta = \theta \).

• For larger angles, a more exact approximation yields

\[
\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots
\]

• Classical aberrations of optical systems can be traced to the behavior of the third order term in the expansion in a raytrace.

• These 5 classical aberrations -- spherical, coma, astigmatism, distortion, and field curvature were enumerated by von Seidel and bear his name.
Quantifying Aberrations – Defining Surfaces

- Actual calculation of ray paths requires the mathematical definition of a surface.
- Spheres are easy...
  - In two dimensions

\[ y^2 + z^2 = R^2 \quad \text{and} \quad z^2 - 2zR + y^2 = 0 \]

- More general conic sections are expressed by

\[ (1-e^2)z^2 - 2zR + y^2 = 0 \quad \text{and} \quad e^2 = \frac{(a^2-b^2)}{a^2} = -K \]

- K is the conic constant (0=circle, 1=parabola, >1=hyperbola, 0-1 = ellipse)
Quantifying Aberrations – Defining Surfaces

• Solving the quadratic parameterizes \( z \) as a function of \( y \) – the shape of the surface as a function of ray height

\[
(1-e^2)z^2 - 2zR + y^2 = 0
\]

\[
z = \frac{R}{P} \left[1 - \sqrt{1 - P \left(\frac{y}{R}\right)^2}\right]
\]

\[P = (1-e^2)\]

• which can be binomial expanded to

\[
z \approx \frac{y^2}{2R} + \frac{P}{8R^3} y^4 + \frac{P^2}{16 R^5} y^6 + \frac{5P^3}{128 R^7} y^8 + \ldots
\]

• if \( P=1 \) this is the (messy) definition of a sphere

• deviations from spherical shape can be defined in terms of \( P \) (conics) or in terms of multipliers of the higher order even terms in \( y \) (aspheric coefficients).
The simplest surface to describe and manufacture is a sphere.

Significant control of aberrations can be obtained by modifying spherical surfaces.

Nearly perfect systems may be prescribed by using aspheres but they may be impossible to manufacture.

The most common aspheres are conics (parabolas, hyperbolas and ellipses)
General Third Order Aberrations

Expanding beyond the linear approximation give the third-order Seidel aberration terms. The angular aberrations are:

\[ AA = a_s \frac{y^3}{R^3} + a_c \frac{y^2 \theta}{R^2} + a_a \frac{y \theta^2}{R} + a_{fc} \frac{y \theta^2}{R} + a_d \theta^3 \]

- coma
- astigmatism
- distortion
- spherical aberration

\( AA \) = angular aberration (e.g. arcsec or radians)
\( a_i \) = constants
\( R \) = radius of curvature
\( y \) = height of ray
\( \theta \) = angle of incidence of rays from object

Taking \( R = f_n y \) then:

\[ AA_{max} = \frac{a_s}{f_n^3} + \frac{a_c}{f_n^2} + \frac{a_a}{f_n} + \frac{a_{fc}}{f_n} + a_d \theta^3 \]

- coma
- astigmatism
- distortion
- spherical aberration
- field curvature

\( AA_{max} = AA \) for \( y = y_{max} = D \)
\( f_n \) = f-number
\( D \) = aperture diameter

Note: the “faster” the optical system, the greater the aberrations!
Spherical Aberration

- An “on-axis” aberration which arises from different radial zones on a optic producing a focus at different distances.
- By its geometrical definition, a parabola is free of spherical aberration (but guilty of others).

**Figure 6K.** Spherical aberration of a concave spherical mirror.
Spherical Aberration

- Spherical aberration can be quantified in three ways.
  - **Transverse** spherical aberration -- the ray height of the most offensive ray at the paraxial focal plane
  - **Longitudinal** spherical aberration -- the separation between "worst" and paraxial focus on the optical axis
  - The diameter of the **circle of least confusion** – possibly of greatest interest to an observer.
Transverse Spherical Aberration

- Transverse SA is measured in terms of the most deviant ray height at the paraxial focus.
- The equation gives the TSA for a mirror of radius, $R$, with conic constant, $K$, as a function of ray height, $y$, from the optical axis.

$$TSA = -(1 + K) \frac{y^3}{2R^2} - 3(1 + K)(3 + K) \frac{y^5}{8R^4}$$
Recognizing Spherical Aberration

Figure 1
Recognizing Spherical Aberration

HUBBLE SPACE TELESCOPE
FAINT OBJECT CAMERA
COMPARATIVE VIEWS OF A STAR

BEFORE COSTAR

AFTER COSTAR
The power of optical design is illustrated by the control of spherical aberration provided by altering lens shape (a.k.a. “bending”).

All of the illustrated lenses have the same focal length.
**Coma**

- Coma arises when incident rays are not parallel to the optical axis/normal.
  - Like spherical aberration, coma is manifested by different radial zones in the optic.
  - Unlike spherical aberration, the images produced by the zones are in sharp focus at the paraxial focal plane.
- Each pair of symmetric points in each radial zone produces a sharp image, but since the lateral magnification is different for each pair each ring of incident rays forms an offset ring producing the classic “comma” image.
Coma

Fig. 7.8 Paraxial image of specific zones when coma is present. Introduction to Lens Design (Geary)

Fig. 7.7 Coma: meridional plane (top); spot diagrams (center); ray fan plots (bottom). Reprinted with permission from Raven and van Venrooij, Telescope Optics (Willmann-Bell, 1988). Introduction to Lens Design (Geary)
Coma in Practice
In a simple lens spherical aberration and coma cannot be minimized simultaneously (but close)

- The optimal shape is close to plano-convex
- but not that this is different from convex-plano ... direction matters!
Astigmatism

- Coma was described in terms of zonal/axial symmetry
- Astigmatism addresses the broken symmetry introduced from the off-axis perspective
  - The optical axis and “chief ray” define a plane – the tangential plane.
  - Ray fans in and parallel to this plane behave differently than ray fans lying in and parallel to the perpendicular “sagittal” plane
  - In particular, the two planes focus at different distances producing sharp perpendicular “line” images at two depths with a circle of least confusion in between.

http://www.vanwalree.com/optics/astigmatism.html
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Astigmatism

• Astigmatism (like coma and SA) is signed. Combining under and over corrected elements can lead to (imperfect) cancellation of the aberration.

• The goal in this cancellation is making the sagittal and tangential planes coincident.
Curvature of Field and Distortion

Fig. 7.11 Meridional plot illustrating field curvature. Reprinted with permission from Rutten and van Venrooij, Telescope Optics (Willmann-Bell, 1988).

Fig. 7.12 Meridional plot illustrating distortion. Reprinted with permission from Rutten and van Venrooij, Telescope Optics (Willmann-Bell, 1988).
Field Flattening

- Curvature of field can be corrected by adding an element directly ahead of the focal plane which serves to introduce a "delay" in focus as a function of distance from the optical axis.
- Little additional aberration can be introduced since each point source "beam" sees a locally flat optic.

Fig 14.8 *Focus shift introduced by a parallel plate in a converging beam.*

Fig 14.10 *Idealistic field flattener.*
Chromatic Aberration

- A lens' focal length depends on the refractive index of the lens material.
- Refractive index (both fortunately and unfortunately) is a function of wavelength.
- Only one wavelength can be exactly in focus at a time.
- Imaging systems often function over broad bandpasses (e.g. K-band spans 2.0 – 2.4 um)
- Optical design mixes materials (e.g. crown and flint glass in a traditional achromat) to mitigate chromatic aberration.
- All-reflective optical systems cannot suffer chromatic aberration.
Chromatic Aberration

- The lensmaker's equation quantifies the difference in focus as a function of wavelength.
- Longitudinal chromatic aberration depends strongly on the dispersion of the material $dn/d\lambda$

$$\frac{1}{f_\lambda} = (n_\lambda - 1) \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Fig. 21.12 Longitudinal Chromatic Aberration for a Simple Lens.
Controlling Chromatic Aberration

- Split the lens into two components (use additional surfaces to control classical aberrations).

- Make lenses out of materials with different dispersive properties
  - The Negative lens has a higher refractive index to control spherical aberration (compensating for its weaker power).
  - Zero spherical aberration can be achieved at only one color. The same is true of chromatic aberration.
  - The doublet is far from perfect and some are more perfect than others.

Fig. 6.5 Achromatic Doublet Designs.
Controlling Chromatic Aberration

- Split the lens into two components (use additional surfaces to control classical aberrations).
- Make lenses out of materials with different dispersive properties
  - The Negative lens has a higher refractive index to control spherical aberration (compensating for its weaker power).
- Zero spherical aberration can be achieved at only one color. The same is true of chromatic aberration.
- The doublet is far from perfect and some are more perfect than others.
The uncorrected chromatic aberration in a double achromat is called “secondary spectrum”.

It can be minimized by appropriate material choices which

- maximize the difference in Abbe number (inverse of broadband dispersion) between the two materials
- minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)

\[ \nu_d = \frac{n_d - 1}{n_F - n_C} \]

Abee Number

\(\nu_d\) = Abbe Number

**Fig. 6.6 Color Curve for a Normal Visually Corrected Doublet.**
Refractive Indices and Dispersions

Fig. 7-6. Refractive index values. Irtran 6 is no longer manufactured by the Eastman Kodak Co. Irtran® is a registered trademark of the Eastman Kodak Co.

Fig. 7-7. $dn/d\lambda$ versus $\lambda$ for selected materials. Irtran 6 is no longer manufactured by the Eastman Kodak Co. Irtran® is a registered trademark of the Eastman Kodak Co.
The uncorrected chromatic aberration in a double achromat is called “secondary spectrum”.

- It can be minimized by appropriate material choices which
  maximize the difference in Abbe number (inverse of broadband dispersion) between the two materials

- minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)

\[
\begin{align*}
P_{FD} &= \frac{n_F - n_D}{n_F - n_C} \\
P_{DC} &= \frac{n_D - n_C}{n_D - n_C}
\end{align*}
\]
The uncorrected chromatic aberration in a double achromat is called “secondary spectrum”.

- It can be minimized by appropriate material choices which
  - maximize the difference in Abee number (inverse of broadband dispersion) between the two materials
  - minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)

Fig. 18.2 Partial dispersion plot for BK7 and SF2.
The uncorrected chromatic aberration in a double achromat is called “secondary spectrum”.

It can be minimized by appropriate material choices which maximize the difference in Abbe number (inverse of broadband dispersion) between the two materials

minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)
The uncorrected chromatic aberration in a double achromat is called “secondary spectrum”.

- It can be minimized by appropriate material choices which
  - maximize the difference in A Bee number (inverse of broadband dispersion) between the two materials
  - minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)
Controlling Chromatic Aberration

The spot diagrams at right represent three different achromatic doublet objectives of varying quality. Today's Cadillac of refractors contain a fluorite element ($$$).

Note that optics designed for the human eye have only weak chromatic constraints.

Fig. 6.10 Spot Diagrams for Various 200 mm Refractor Objectives.
Chromatic Aberration Correction

Fig. 19.2 The color correction path.
Classical Optical Design

- With raytracing/computers as a tool, a designer can leverage the many degrees of freedom in a system to minimize image aberration.
- Degrees of freedom include:

  - Optical Materials
  - Reflective vs. Refractive
  - Number of surfaces/lenses
  - Curvature of surfaces
  - Lens thickness
  - Lens Separation
  - Custom Surfaces (aspheres)
  - Stops (although not usually in Astronomy)
  - Tilts

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