ELECTROSTATIC FORCE AND COULOMB’S LAW

OBJECTIVES

- Review Coulomb’s law.
- Verify distance dependence of Coulomb’s law.
- Verify charge dependence of Coulomb’s law.
- Investigate whether charge is equally distributed on two identical bodies in contact.

OVERVIEW

The fundamental relation expressing the force $F$ acting between two electric charges, $q_1$ and $q_2$, and separated by a distance $r$ is given by Coulomb’s law:

$$ F = \frac{kq_1q_2}{r^2} $$

It is interesting to notice that this expression is formally similar to Newton’s law governing the gravitational attraction between two massive bodies, although there are some fundamental differences. To start with, it has been established that the electrostatic force is orders of magnitude stronger than the gravitational force. In spite of this, the effects of the electrostatic force are not easily observed at the macroscopic level, in contrast with the easily observed effect of the gravitational pull. The main reason for this is that electric charges come in two types (so called “positive” and “negative”). When felt by a third object, the forces due to two equal but opposite charges in close proximity (like a hydrogen atom) cancel each other. Gravitational masses, however, come with one sign only (we have never observed an object carrying a “negative” mass), gravitational pulls of individual elementary particles forming a large body add up, and no object intervenes to cancel the gravitational force of a given body. Even so, the electrostatic force plays a most fundamental role in shaping the world as we know it; it is responsible for holding the negative atomic electrons around the positive nucleus. The electrostatic force governs the whole spectrum of chemical and, consequently, biological processes.

In this experiment, you will verify the validity of Coulomb’s law by measuring the force acting between two charged spheres. The force
will be measured by means of a torsion balance: in your setup you will produce a repulsion between a fixed sphere and another one mounted on a rod, counter-balanced and suspended from a thin torsion wire. To perform the experiment both spheres are charged and kept at a given distance. The electrostatic repulsion between the spheres causes the torsion wire to twist, the twist angle being proportional to the force acting upon the sphere, \( F \sim \theta \).

To deposit a certain amount of charge onto the spheres, you will use a power supply, an instrument capable of providing a stable voltage at one of its terminals. By touching the spheres with the power supply output probe, the sphere will be brought to the same voltage as the supply output, and a certain amount of charge will be deposited onto the sphere. You might already have learned, or will learn soon, that the amount of charge \( q \) required to bring a body to a certain \( V \) is proportional to the voltage itself, i.e.

\[
q = CV
\]  

where the constant of proportionality \( C \) is called capacitance, which is a characteristic property of any given body.

One can obtain an intuitive appreciation of the meaning of capacitance by thinking in hydrostatic terms: if you have two containers, a slim one and a fat one, filled to an equal height, the liquid at the top has the same (gravitational) potential energy for either container. At the same time, more liquid is required to fill the fat container to the desired height: the fat container has more capacity… If you replace the quantity of liquid with amount of charge and the gravitational potential with electrical potential, the comparison stands.

Experiments with the Coulomb balance could, in principle, be quite accurate; yet, as with any quantitative electrostatic experiment, spurious effects can intervene to spoil the measurement. A charged shirt sleeve, an open window, an excessively humid day - any of these effects and many more can affect your experiment. For your setup the following precautions should be taken:

- The experiment should be performed during part of the year when the humidity is low. The winter is an ideal time to do this experiment. We will have the air conditioners and dehumidifiers on prior to your starting this experiment. However, we must turn them off when you start the experiment to avoid air currents, which have been shown in the past to disturb the performance of the torsion balance.

- The experiment should be performed in a draft free room, and the equipment installed on an insulating table and away from
the walls. (Were any conducting surface near your equipment, image charges would arise in it, with the effect of distorting your measurements.)

BOOKS, COATS, ETC. MAY ADVERSELY AFFECT YOUR MEASUREMENTS. MAKE SURE THEY ARE NOT ON THE TABLE!

YOU MAY ALSO WANT TO OCCASSIONALY TOUCH BOTH HANDS TO A GROUND (LIKE ON THE POWER SUPPLY) TO MAKE SURE CHARGE DOES NOT BUILD UP ON YOUR BODY.

APPARATUS

- Pasco Coulomb Balance apparatus
- Pasco 6 kV power supply with sphere charging probe

A photo of the Pasco coulomb torsion balance apparatus is shown in Figure 1 and a schematic diagram in Figure 2.

Figure 1. Torsion Balance Apparatus. The two spheres are near each other in the center. The left one slides back and forth to the left and right. The torsion fiber is vertical, but is too small to be seen. The right sphere rotates back and forth depending on the attraction to the left sphere.
Figure 2. Torsion balance assembly. a) Side view. b) Top see-through view (the torsion knob assembly is shown with dotted lines).
INVESTIGATION 1: FORCE VS. DISTANCE

ACTIVITY 1-1: SET UP TORSION BALANCE ASSEMBLY

The torsion balance assembly is shown schematically in Figure 2, which gives a side and a top view. Before starting your measurements make sure the apparatus is set up properly. Follow the directions below very carefully. If you have questions, consult your TA.

1. Set the torsion knob on the top of the balance to be aligned with the zero degree mark.

2. The counterweight vane balancing the suspended sphere should be fairly horizontal. The mark at its center should be aligned with the corresponding mark on the index arm. If they are not aligned, follow the directions below carefully. Ask your INSTRUCTOR if you have any uncertainty as to what to do. Find the knob associated with the torsion wire retainer at the bottom of the balance. THIS KNOB SHOULD NEVER BE UNSCREWED. IF IT IS, THEN TERRIBLE THINGS WILL HAPPEN TO YOUR EXPERIMENT AND YOUR GRADE! ROTATE (not screw in or out!) in the plane of the table the knob until the mark on the counterweight vane is aligned with the corresponding mark on the index arm. If you are not sure what rotate means in this regard, ask your TA, because we agree there might be some confusion.

3. The mark on the torsion knob at the top of the balance should be aligned with the zero degree mark.

4. There are two charcoal grey spheres that will be charged in this experiment. One is located on the counterweight vane, and the other is mounted on the sliding guide. Avoid touching the rods supporting the spheres: fingerprints produce a conductive coating, which causes charge leakage. Never touch the spheres themselves!

Figure 3. Complete Coulomb balance assembly including torsion balance and independent sphere on sliding track.
5. Slowly slide the second sphere mounted on the sliding guide as close as possible to the suspended sphere (see Figure 3), the spheres should be aligned, both vertically and laterally.

6. When the two spheres are barely touching, the pointer on the sliding arm should read 3.8 cm on the centimeter scale (given that 3.8 cm represents the diameter of the spheres, this will ensure that the reading on the centimeter scale will reflect the distance between the centers of the spheres).

7. If any of the above conditions is not met, ask your INSTRUCTOR for assistance.

**CAUTION:**
The whole torsion balance assembly is extremely delicate. You should avoid handling it as much as possible. In particular, you should be careful not to hit the hardly visible wire to which the balanced sphere is suspended.

---

**ACTIVITY 1-2: CHARGING SPHERES**

Now we will discuss charging the two spheres. **You must pay careful attention to avoid receiving an electrical shock.**

1. Refer to Figure 4 for the Kilovolt Power Supply. It is absolutely essential that you turn the High Voltage Adjust knob down to zero (CCW) before turning on or off the power supply. **If not, serious injury may result.** Black wires should be connected to the negative side of the 6 kV output. One black wire goes to ground and the other black wire will be used to discharge the two spheres when instructed. The red wire is the one that will be at high voltage and used to charge the two spheres. Hold the red wire by the plastic handle; **never touch the metal probe at the end of the red handle.**
2. Now let’s practice charging and discharging the spheres. Move the two spheres as far apart as the slide assembly allows. Then grab the black wire near its end and touch both spheres for a couple of seconds each with the metal end to discharge them.

3. While holding the plastic handle of the red wire, check that the high voltage adjust knob is down to zero (CCW). Turn the power supply on and use the high voltage knob to adjust the voltage to 6 kV. While holding the red handle touch both spheres for a couple of seconds to charge the spheres, then immediately turn the voltage down and the power supply off. Then place the red probe on the table. When charging the spheres, hold the charging probe near the end of the handle (i.e. far from the metal tip), so that your hand is as far from the spheres as possible. The high voltage at the terminals of the supply can cause leakage currents, which will affect the torsion balance. Place both ends of the wire leads far away from the torsion balance apparatus.

4. When performing the measurements, stand at the maximum comfortable distance away from the torsion balance. This will minimize the effects of static charges that may collect in your clothing. The effect is particularly severe if you are wearing synthetic fabrics.

5. To minimize the effect of slow unavoidable charge leakage into the air, perform measurements as quickly as possible after charging. Recharge the spheres before each measurement.

6. Now discharge the two spheres by touching them both for a couple of seconds with the metal end of the black wire. Place both wire leads far away from the torsion balance assembly.

**Activity 1-3: Data Taking**

1. In order to start from a known situation, move the sliding sphere as far as possible along its track from the suspended sphere. Then fully discharge the two spheres by touching them with the grounded probe (end of black wire). With the spheres at maximum separation, turn on the power supply, increase the voltage to 6 kilovolts (kV), and charge both spheres by touching them with the red wire probe. Turn down the voltage on the power supply immediately after charging the sphere. Place the probe far away from the spheres. Remember that time is of the essence here. You need to move promptly!

2. Position the sliding sphere at 20 cm. Notice that the suspended sphere will tend to move away from the other sphere, because it is under the effect of the electrostatic repulsion. This will cause the whole suspended assembly to rotate away from its equilibrium position. The rotation angle of the assembly is proportional to the force acting upon the sphere. When equilibrium is reached the torsion of the wire
counterbalances the repulsive force. Adjust (slowly) the torsion knob on top of the balance as necessary to bring the pendulum back to the zero position. Record the angle below in Table 1-1.

**HINT: IN A HUMID ROOM, CHARGE WILL BE LEAKING OFF THE SPHERES CONTINUOUSLY, SO TIME IS OF THE ESSENCE! DO NOT DALLY! YOU MAY HAVE TO START OVER.**

3. You should then repeat the above procedure to convince yourself that you can achieve reproducible results: discharge the spheres, check the zero of the torsion balance, then separate the spheres to their maximum distance, recharge them to the same 6 kV voltage, and reposition the sliding sphere to the 20 cm distance. Measure the torsion angle and record it in Table 1-1. Repeat the procedure a few times, and verify that the torsion angle is reproducible to 10% or better. Three measurements that agree are certainly enough. The torsion balance zero should be stable; you may want to occasionally check it, but you do not need to check it every measurement.

**Table 1-1 Torsion Angle Measurements**

<table>
<thead>
<tr>
<th>Distance</th>
<th>Torsion angle (degrees)</th>
<th>Mean angle value (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>XXXXXXXXXXXX</td>
</tr>
</tbody>
</table>
4. Repeat the procedure for at least four other values of the separation between the spheres, e.g. for 16, 13, 10, 8 cm. For each separation measure the angle two or three times, and calculate the mean value of the angle for each setting and put that in the appropriate place in the 3rd column of Table 1-1. If you need more than 3 measurements for each distance, list your extra measurements out to the right side. Once you get the hang of it, two measurements that agree to 0.5° should be enough.

**Question 1-1:** We keep mentioning that charge may be leaking off the charged spheres. When you came into the lab room, the air conditioners may have been on as well as a dehumidifier. We find the humidity is higher in the summer than in the winter. We tend to get shocked in the winter if we are walking across a carpeted room and touch a light switch. What is this all about? Why is humidity important? Explain.

---

**Activity 1-4: Verifying Distance Dependence**

1. We now want to verify the $1/r^2$ dependence of the force in Eq. (1). Open the file **L2.A1-4 Verifying Distance Dependence** in Excel and enter your data for the distance and mean angle torsion values from Table 1-1 into the yellow regions of the Excel file. Because the force is directly proportional to the angle, you can use the angle to indicate force.

**Question 1-2:** We have stated that the force is directly proportional to the angle of rotation. Is it possible to determine the absolute value of the force? Explain.

2. The Excel file will do the calculations for you. Note that you enter the distances between the sphere centers in cm. Excel then has a column where $1/r^2$ is calculated in m$^{-2}$ (that is cm is converted to meters). A plot is shown of the force (angles) versus the inverse...
square of the distance. Fit the data with a linear line. Show the trendline, \( R^2 \) (indication of goodness of fit) and the linear fit parameters on the plot. Write your values below:

Linear fit: ___________________________

\( R^2 \): _______________

**Question 1-3:** Are your data well fit with the linear line? If not, do you have any suggestions what the problem might be? Explain.

**Question 1-4:** What value should the force have when the distance \( d \) is very large? According to your linear line fit, what value does your data give for this force? Is this reasonable? Explain.

Force value at large distances: angle ___________

3. You should notice a nearly **linear** behavior, with the deviations from linearity being greater for the smaller values of the distance (= larger values of \( 1/r^2 \)). The reason for the deviation is that, at short distances, the charged spheres cannot be thought anymore as point charges. A charged conductive sphere, if it is isolated from other electrostatic influences, will act as a point charge (see Fig. 5a). The charges distribute themselves evenly on the surface of the sphere, so that the center of the charge distribution is just at the center of the sphere. However, when two charged spheres are separated by a distance that is not large compared to the size of the spheres, the charges will redistribute themselves on the spheres so as to minimize the electrostatic energy (Fig. 5b). The force between the spheres will therefore be less than it would be if the charged spheres were actually point charges.
Electrostatic Force and Coulomb’s Law

Figure 5. a) The charged conducting spheres when separated by great distance have their charges equally spaced on the sphere. b) However, when the two spheres are brought closer together, the charges repel each other and are not equally spaced throughout the surface of the sphere.

4. In more advanced physics courses using calculus, a correction factor $B$ is determined to correct for the redistribution of charge. You will multiply each $\theta$ by the factor $1/B$, where

$$B = 1 - 4 \frac{a^3}{d},$$

and $a$ equals the radius of the spheres and $d$ is the separation distance between the center of the spheres. You were given earlier that the spheres had a diameter of 3.8 cm. Enter the value of $a$ in cm in the yellow shaded cell of I3. Then Excel will calculate in column E the correction value of $B$ at each distance $d$. In column F, the mean values of $\theta$ are multiplied by $B$. 

5. Do not erase any of your original data or plots in Excel. Note that another plot of the corrected angle (force) versus $1/r^2$ or $1/d^2$ in our case is shown. Fit it with a linear line and show your values on the new (2nd) plot. Write your fitting values below.

Linear fit: _______________________

$R^2$: ________________
**Question 1-5:** Is your data better fit with the correction? What about the value of the force when the spheres are very far apart? What is this value now and is it better than the uncorrected data?

Force value at large distances: angle ________

**Question 1-6:** If your corrected data is not better than the uncorrected data, do you have any suggestions what the problem might be? Explain.

6. Print out all the Excel **data and plots** and include in your report. Try to fit it into one page.

**Question 1-7:** The charge on the spheres will redistribute itself as the spheres are brought closer together. Will this increase or decrease the force (at a given distance) from that expected if the charge were equally placed on the spheres? Explain.

**Question 1-8:** What would be the effect in the previous question if the spheres had equal, but opposite charges? Explain.
INVESTIGATION 2: FORCE VS. CHARGE

In the second part of your lab, you will verify the dependence of the force upon the charge, $F \sim q_1q_2$. As you do not actually perform a measurement of the charge deposited upon the spheres, you should remember that the charge is proportional to the voltage (Eq. 2), therefore you can express your results in terms of voltage applied to the spheres vs. the resulting force. There are two ways we can perform this experiment as we will read in Activities 2-1 and 2-2. Do not take any data until Activity 2-3.

ACTIVITY 2-1: FIRST PROCEDURE

1. The first procedure consists of charging the suspended sphere at a constant voltage (e.g. 6 kV) and performing a series of force measurements for different values of the second sliding sphere voltage (e.g. 1, 2, 3, 4, 5, 6 kV).

2. The measurements should be performed with the two spheres always at the same distance (e.g. 10 cm), although you should remember that, when charging the spheres, they should be kept at their maximum separation.

3. The linearity of the force vs. charge can be verified by plotting the deflection angle vs. the voltage of the sliding sphere. A possible drawback of this procedure is that the suspended sphere, originally charged at 6 kV, can dissipate some of its charge throughout the execution of the measurement.

Question 2-1: Can you think of a procedure that would avoid the potential error discussed in step 3? If so, explain.

Question 2-2: Would it be prudent to make the correction for $B$ again? Explain.
ACTIVITY 2-2: SECOND PROCEDURE

1. In the second procedure, you perform your measurements with both spheres charged at the same potential (1, 2, 3, ... kV). This way you have a better control upon the sphere charge, but you should be careful how to plot your data. (Remember, the force is proportional to the product of the charges.)

2. The measurements should be performed with the two spheres always at the same distance (e.g. 10 cm), although you should remember that, when charging the spheres, they should be kept at their maximum separation.

Question 2-3: How should the data be plotted in this procedure?

Question 2-4: Which of the two procedures for finding the force dependency on charge do you think would be quicker? Explain.

Prediction 2-1: In order to obtain good results for either procedure, what distance between the balls do you think is best? Explain. Do this before coming to lab.

ACTIVITY 2-3: DATA TAKING AND ANALYSIS

We will perform the experiment only according to the second procedure for the reasons that you hopefully stated in Question 2-4.
1. Put your data into Table 2-1 and into an Excel file of your design or use L2.A2-3ChargeDependence.2nd procedure.nodata.xls. Make sure all the columns in Excel are well indicated, and that another person can understand what you have done. Indicate all data values and measurements in your Excel file.

<table>
<thead>
<tr>
<th>Table 2-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (cm)</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

2. Make a plot of your data indicating how the force changes with the product of the two charges. Arrange the data so that a linear line fits your data. Show the fits and fitting parameters on the plot.

3. Print out your data and plots for your group report. List below the fitting equation that you obtained.

   Linear fit: ___________________________

   $R^2$ value: __________

**INVESTIGATION 3: CHARGE DISTRIBUTION AMONG IDENTICAL BODIES**

Suppose you have two identical conductors, like the two charged spheres in the present experiment. The first sphere carries a certain charge $q$, the second being free of charge. If you bring the two bodies into contact, a certain amount of charge will migrate from the charged body to the other one. If the two bodies are identical, in the final configuration the charge will be equally distributed among the two bodies, i.e. each body will carry a charge $q/2$.

[If you remember about capacitance, you could easily prove this by remembering that $q = CV$, two identical bodies will have the same value of capacitance $C$, and, when brought into contact, the two bodies will reach the same potential. The hydrostatic model can be of help again: if you have two identical containers, one filled to a given level,
the other empty, when brought into communication they will both end up filled at half the original level.]

**Prediction 3-1:** List below the procedure you think should be followed in this investigation. How do you intend to prove that charge is equally distributed over two identical conductors in contact? Do this before coming to lab.

1. In practice, what you need to do is to bring the two spheres to the same potential, measure the force at a given distance. Discharge one of the spheres, bring it into contact with the second sphere and re-measure the force at the same distance. Do this at least three times. Show your data below.

2. Write carefully below **precisely** what you actually did for this experiment. Be sure to include all your data and be sure your Instructor can follow what you did. Show your calculations to prove that charge is equally distributed over two identical conductors in contact.

**Question 3-1:** Discuss how well you have shown that charge is equally distributed over two identical bodies in contact. How well do your results agree with what you expected? Discuss whether this agreement is sufficient and give reasons.